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1898 to 1922
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TO

The Very INGENIOUS

Charles Du-Bois, Eſq;

OF THE

City of LONDON.

SIR,

A Perſon who comes upon the Account that I do; to return Thanks for Kindneſſes out of the common Road; to have anſwer'd his Deſign the more effectualy, ſhou'd have found ſome other Way for't than this. A Man ſeems but poorly to expreſs his Sence of Favours that are altogether extraordinary; when the Me-

2-29-40 M.H.M.

thod he takes to do it in, has the *little Character* of being a *vulgar* and *cuſtomary* One. Such is a Dedication: A Thing written *every Day*; by *every Pen*; and to *every Perſon*: The ſtanding Compliment, that paſſes upon all Occaſions; and that which inſolvent Debtors (thoſe that wou'd be juſt) have ever made a Trade of Offering, inſtead of Payment. One therefore that is conſcious of ſo great a Debt (as I wou'd be thought to own by this Addreſs) muſt neceſſarily lie under a mighty Diſadvantage. For neither great Obligations on the one Hand, nor the Acknowledgments of a grateful Mind on the other, can (by any Painting here) ever appear to be what they are. A Man may deſcribe the Favours of a generous Friend, and his own juſt Sence and Reſentments of them too: But after all; be they ever ſo great in themſelves, they leſſen by coming down into *this Way of Repreſentation*. Notwithſtanding this, the *Great Law of Gratitude* urges me to obey it: And I have concluded, 'tis better to do it with ſome Diſadvantages; than to incur the Guilt of a total Neglect. Be pleaſed then (SIR) to let this ſtand as the Monument of a Gratitude that *wou'd have been great*: A ſmall and trivial Acknowledgment I confeſs: But yet the genuine Product of that, which is allow'd to add a Value to the moſt deſpicable Things: For ſuch is a ſincere good Will. Had there been no Reaſons on my part; and I had conſider'd only thoſe that are to be drawn from your ſelf; this Book had been inſcribed to You of courſe. Every Subject that's fine and curious does of Right belong to You: But *Philoſophical Matters are peculiarly Yours.* In the Midſt of all Your weighty Buſineſſes, You find Time to converſe with Nature, to ſo good Purpoſe; that thoſe that know You ſo well, as 'tis my Happineſs to do, are ſenſible She's fond of Your Acquaintance; and does by very particular Favours bribe You to continue it. And tho' the Liberty I take in aſſerting the Truth may give ſome Umbrage to a *Vertue* (which at preſent ſhall be Nameleſs) yet ſo much do I confide in the Protection of *Your Temper*, that I dare venture to enflame the Reckoning : I'll confeſs the Crime to be voluntary and preſumptuous, and that I knew beforehand, how nice and tender You were in this Point. If You (SIR) think fit to carry on ſuch a Deſign, as that of hiding Beauties, which ought to be expoſed to the World: You muſt pardon thoſe People that act ſo good a Part, as to diſcover the Contrivance, and ſo are free as to ſay, 'tis the only Peice of Injuſtice You can be guilty of. Whatever Vices, Addreſſes of this nature are commonly chargeable with ; mine to You, has Two Vertues to recommend it, Truth and Innocence. For 'tis true in the higheſt Degree, that You have made me Your Debtor ; as You do the Reſt of Mankind Your Admirers.

Nor is it leſs true, that I do with the greateſt Pleaſure ſubſcribe my ſelf,

SIR,

Your moſt Obliged Friend,

And very Humble Servant,

H. DITTON.

THE PREFACE.

IN this little Treatiſe I have endeavour'd to explain that Part of Mr. Newton's Principia, *which is Fundamental to the Reſt of the Divine Book; and particularly to the* Syſtem *of the* World. *And I have all along taken Care to render Things ſo far plain and eaſie, that they may be underſtood by all thoſe that will beſtow ſome Application upon them, and have a competent Share of the neceſſary Furniture, namely* Algebra *and* Conick Sections. *'Tis entirely owing to the ſtupendious Genius and Sagacity of this Author, that we have any ſuch Theorems now extant in the World, as thoſe which his incomparable Book contains. The Difficulties in the Theories of the Heavenly Bodies might have continued a laſting Reproach to the Mathematicks of our Earth, if ſuch a Mind as his had not undertaken to vanquiſh and remove them.*

The Materials that this Book is compoſed of, are ſo abſolutely Mr. Newton's *Property, that I dare hardly pretend to call any thing mine,*

mine.' The Principles moſt certainly are all his own: And if I have attempted any where to make any Uſes of them, or to draw any Conſequences from them; yet the indiſputable Right that he has to the Former, gives him a Title to the Latter alſo, where they are juſt and good. This is certain, that his Inventions are new and compleat; and equally exclude all the Additions and Claims of thoſe that come after. But that I may not be imagin'd to throw my Miſtakes (if I have committed any) upon the Great Author, and by ſaying all is his, either be thought to reflect on him, or to deſign to ſcreen, and ſhelter my ſelf: Whatſoever is amiſs and faulty, I take it entirely upon my ſelf, and declare it not to be the Effect of the Principles that ſhou'd have guided me, but of my own Inadvertency too, and not being guided by them.

And as I ought not to have omitted (or ſaid leſs than) this, upon this Head; ſo the ſtricteſt Juſtice, and the Rules of good Manners, as I apprehend the Matter, did both oblige me to ſay what Intereſt he had in this Book, and how many Ways 'tis his own: For tho' there's no manner of Danger in not owning Mr. Newton*'s Theorems; and his Inventions, in what Hand ſoever they are, will eaſily diſcover and point to their true Author; yet it ſeems at leaſt handſome and deſcent for* *a Man to tell the World plainly when he takes any ſuch Liberties.*

As for the Reception this Undertaking is like to meet with in the World, I have no Uneaſineſs at all about the Matter. Some People cry down all Books of courſe, that are not writ by themſelves, or thoſe of their own Faction: As others do out of pure Spleen, becauſe the ill-natur'd Subject won't ſtoop to their Underſtandings. Some People again naturally love to find Fault; their Genius prompts them to ſnarle and cenſure, and they take the ſame Pleaſure and Satisfaction in that, that others do in the very beſt Employments of their Minds. In a Word, every Man that writes, and every thing that's written, runs the Riſque of a Thouſand Cenſures: As many at leaſt, as there are Prejudices, Humours and Fancies to be met with. The Fates of good and bad Books are very often like thoſe of good and bad Men in this World: So uncertain and confuſed, that the true Characters of either are often not to be known. In Writing as well as in Manners, he that does ill may chance to be commended for't, and he that does well without a juſt Recompence, ought to remember that many a good Thing has been ſerv'd ſo before.

If any Perſon thinks this Work wou'd have become his own Hand better than mine; I'll eaſily yield that Point to him. For tho' a *great many that make a Noiſe and a Buſtle, can do little elſe but that: Yet there are ſome that will boaſt and perform too; ſo unlucky a Lodging have their good Qualities gotten. I deſire only it may be minded, that my Deſign here, is not to entertain thoſe that are of an advanced Standing in this noble Philoſophy, but to introduce thoſe that wou'd be acquainted with it, and have as much Mathematicks as is neceſſary to qualifie them for it. And therefore upon ſuch an Account a Man is obliged to deſcend to a Multitude of little Things, that otherwiſe wou'd not be neceſſary, and may ſeem to take off from the Elegancy of what he does. 'Twou'd be an eaſie Matter, either by omitting at firſt, or afterwards taking off, ſeveral uſeful Superfluities brought in for the Sake of Beginners, to make Things of this Kind appear, in a more pleaſing Dreſs, even to Judges.*

Farther, To render what I have done more univerſally ſerviceable here at Home, I choſe to make it appear in Engliſh *rather than* Latin. *For if it be granted that Mr.* Newton'*s Diſcoveries are but barely uſeful, there's no Reaſon why a Multitude of very capable Minds ſhou'd be debarr'd from them meerly for want of a Language. But if it be granted (as it will be by all that know them) that they are the fineſt Pieces of Human Knowledge* *in the World, then the Reaſon grows very ſtrong, why they ſhou'd be put in ſuch a Language as they underſtand. Again, a Man is bound to believe that the* Illuſtrious Author *deſign'd, what he was pleaſed to publiſh, ſhou'd be known; and that not to a few Perſons only who fortunately had the Advantage of an Education; but to all whom Nature had qualified with good Reaſon, and their own Induſtry with Skill in Mathematicks. To think leſs than this, is to ſuppoſe him capable of envying a Part of Mankind the Knowledge of his Diſcoveries, which wou'd be no ſmall Affront to a Soul much leſs than his. Now if there be any that ſhou'd ſay, 'tis to leſſen and depretiate theſe ſublime Pieces of Philoſophy to expoſe them to the Vulgar by a Publication in* Engliſh: *That they ought to be entruſted only with thoſe that have Skill and Judgment to make a right Uſe of them: And (in a Word) that others, by going away with crude imperfect Notions; will do themſelves no Kindneſs, and diſparage the Things they talk of into the Bargain.*

Thus indeed, I confeſs, do ſome People argue for keeping the Sacred Books in an unknown Tongue: But we pretend to a Proteſtant Liberty, at leaſt with reſpect to our Philoſophy. And methinks both in the one and the other Caſe; 'tis unreaſonable that thoſe that wou'd *make a good Improvement; ſhou'd be kept in Darkneſs, for the Sake of them, that wou'd abuſe their Light. If a curious Treatiſe ſhou'd not appear in the vulgar Tongue of a Country, for fear ſome People ſhou'd play the Fool with it, and make an impertinent Uſe of what it diſcovers: 'Twill be very hard to find what Language it ought to be printed in. For 'tis no unlikely thing that there ſhou'd be Fools of all Nations and Languages, and 'tis Ten to One but ſome of theſe, do meet with an Author, which ſoever of them all he chuſes. We vainly flatter our ſelves, no doubt, to think we have ſecured a fine Thing from vulgar Underſtandings, by putting it into* Latin. *For we ought to believe that there are Mechanicks in Learning, as well as in Trade, and ſome vulgar People that drudge with Books, as well as others with Spades and Hammers. 'Twere well if all Men cou'd get Skill in Languages; but good Senſe and this Skill are very often no Friends to each other, and won't live together in the ſame Head. And when they are apart, methinks, 'tis no hard Point to determine on which Side the vulgar Underſtanding lies. But to anſwer thoſe that won't otherwiſe be anſwer'd; let them conſider in what Language our moſt celebrated* Geometry Profeſſor, Capt. Halley, *has condeſcended to publiſh a good part of his invaluable Performances. And let 'em conſider* Mr. Newton's ton's *Book of Light and Colours, where the rich Treaſures lie expos'd alſo in our own Tongue.*

And now I have done with that Matter: I ſhall only add as to the Book it ſelf, that, I hope, the Peruſal of it will neither be unpleaſant nor unprofitable to the ingenious well qualified Reader. And when he comes to perceive the Uſe and Advantage of theſe Things, and how they tend to lead him into the moſt profound and pleaſing Speculations of Nature; let him then return his Thanks to the great Genius to whom all this is owing; and if he thinks it fit to allow my Pains any little Share of his Acknowledgment, he'll make me his Debtor thereby, and bind me to farther Endeavours for his Service.

But tho' I have treſpaſs'd upon the Reader's Patience thus long, I muſt beg his Pardon, if I deſire the Continuance of it yet a little longer. The Laws *deliver'd in this Treatiſe relating to the* Motions, Forces *and* Velocities *of* Bodies, *are but very particular and narrow Things, in reſpect of that great and univerſal* Law *of the World, that* Motion is eſſential to Matter. *The Perſon that advances this (and paſſes his Word for the Truth on't) is Mr.* Toland, *who has written a large Letter about it, which may be ſeen in his Book,* *entituled,* Letters to Serena. *And as all the great Laws of Nature are capable of many excellent and profitable Applications; ſo the Uſes of this Principle are (if we dare believe him) both very many and very great. It's a Clue that will faithfully guide a Man thro' ſome of the moſt perplex'd Difficulties in Nature, and enable him to give ſuch Solutions, as cannot be given any other Way. Mr.* Toland *ſays as much as this amounts to of it; which is the ſame Thing, as if he had ſaid in other Words, That 'twill conduce to the making* Philoſophy *as little* myſterious *as* Chriſtianity *it ſelf. How little that is, he, perhaps, very well knows; and how little Darkneſs this Notion leaves in one particular Point in Phyſicks, without any thing of a* Perhaps, *I am ſure I do know. For to give a clear and ſatisfactory Account of the* Vis Motrix *and the* Vis Impreſſa; *he ſays, That the* Vis Motrix *is the general and eſſential Action of Matter; and the* Vis Impreſſa *the particular Determination of that general Action. Now, this makes all the* Myſtery *of the Buſineſs vaniſh at once, and the Philoſophers need torture their Heads about it no longer. In a Word, other People may think as they pleaſe, but I muſt be excuſed, if I pretend to believe, that Mr.* Toland *is in Jeſt, and acts a Part. 'Tis much more civil to ſuppoſe this, I am ſure, than to believe him to be in Earneſt. For* *when a Man appears to be warm and ſerious in the Management of a Subject, talks a vaſt deal upon it, ſpeaks with the laſt Degree of Aſſurance, and, after all, ſays nothing to the Purpoſe: To ſay, He believes what he ſays, is to give him the ill Name of a Fool, and to call him ſenſleſs and ſtupid; which are Terms I wou'd uſe to no Man. But to take that Part which I do, is at moſt but to make him a little Trickiſhly diſpoſed, and to love (in a merry Way) to put upon Peoples Underſtandings; which Character, perhaps, may ſeem leſs diſpleaſing and diſagreeable to Mr.* Toland, *than the former. In Confidence of this, I endeavour to juſtifie my Surmiſe by the following Reaſons.*

1. *Becauſe he takes particular Care to uſe no Argument, but ſuch as he is ſure not to conclude with: And if he does pretend to prove his Point, 'tis in ſuch a Manner as can never poſſibly perſwade the Reader to be of his Mind.*

2. *When he takes Notice of any thing that may be an Objection in his Way, he deals with it in a very Gentleman-like Manner; ſoftly and tenderly, without making any rude Aſſault, or offering the leaſt Violence to it.*

3. *He is no leſs certain to baulk his Reader's Expectation, than he is careful to raiſe it: And after he has made very liberal Promiſes, does neither perform them, nor ask Pardon for the Omiſſion.*

4. *He oppoſes very groundleſs precarious Notions to Truths that are ſufficiently proved, and lays his own Opinions without any Reaſons, in the Ballance with Truths that are capable of Demonſtration.*

And when a Man does ſuch Things as theſe, 'tis not unreaſonable to ſuſpect, that he is either in Jeſt, or out of his Wits: But the firſt Suppoſition is the more good-natur'd and gentile of the Two, and therefore I rather make that than the other.

Let us now briefly attempt the Juſtification of theſe Reaſons alledged. And in order to the firſt, we may obſerve, that this Gentleman expreſſes his Mind about Motion's being eſſential to Matter with the utmoſt Plainneſs and Perſpicuity; ſo that 'tis impoſſible to doubt of or miſtake his true Meaning: I hold, (*ſays, he* Pag. 159.) that Motion is eſſential to Matter; that is to ſay, as inſeparable from its Nature as Impenetrability or Extenſion, and that it ought to make a

Part of its Definition. *And* (Pag. 161.) My only Buſineſs is to prove Matter neceſſarily active, as well as extended. *And, to quote no more, he tells us* (Pag. 168.) He maintains, that Matter can be no more conceiv'd without Motion, than without Extenſion, and that the one is as inſeparable from it as the other. *Now, after he has (in* Letter V.) *ſpent ſeveral Pages in diſcourſing upon the uncertain and changeable State of Matter every where in the Univerſe; ſhewing, that the Parts of it undergo a Thouſand Viciſſitudes and Alterations, and this in Bodies of all Forms and Textures, even thoſe which we ſuſpect the leaſt liable to ſuch Changes. Having done this, he comes to gather up all into a Concluſion, and infers after this manner* (Pag. 202.) I think, after all that has been ſaid, I may now venture to conclude, that Action is eſſential to Matter; ſince it muſt be the real Subject of all thoſe Modifications, which are call'd Local Motions, Changes, Differences or Diverſities: And principally, becauſe abſolute Repoſe (on which the Inactivity or Lumpiſhneſs of Matter was built) is entirely deſtroy'd, and prov'd no where to exiſt. *Now all this is ſo perfectly Foreign to Mr.*Toland'*s Purpoſe, that out of* 1000 *People that ſhou'd grant him the Truth of his Premiſes,* 999 *at leaſt wou'd deny that his Concluſion* *follow'd from it. Grant that the Parts of Matter every where throughout the Univerſe (tho' his Proofs do not pretend to reach ſo far) were in this State of continual Flux and Change. Grant that they undergo all the Removes and Shiftings of Place and Form that he ſpeaks of, and Ten Thouſand more. Let it be taken as a Truth, that there is no abſolute Repoſe, but Motion in all Syſtems of Matter, both greater and leſs: And after all, I ſay,* Quid Colligis inde? *Will it follow, that this Action is of the Eſſence of Matter, and that Matter can't be Matter without it? By no Means in the World. Mr.* Toland *can't but know, that 'tis no good Arguing to ſay, this Thing is ſo in Fact, therefore 'tis naturally, neceſſarily, eſſentially ſo, and impoſſible to be otherwiſe. I find this Thing may be predicated of this Subject, and therefore the Subject can't exiſt, can't be defin'd, can't be conceiv'd of without it. If this were allowable, a Man might infer a Multitude of Things that are notoriouſly falſe and abſurd, and which all Mankind will grant to be ſo, at the firſt Hearing. And therefore I wou'd hope I need not queſtion but he was aware of it, and took this Method of proving only to ſhew, that, however he was pleaſed to banter, he did not believe what he ſaid to be true.*

In the next Place, as to his Way of treating Objections, we ſhall find him ever curiouſly obſerving the Motions of a Difficulty towards him, and by prudent and well contriv'd Steps getting out of the Reach on't. For Inſtance, The Buſineſs of the different Specifick Gravities of Bodies equal in Bulk, being urged to prove a Vacuum, *from whence the Inactivity of Matter wou'd be fairly inferr'd. Mr.* Toland *takes Notice of it* (Pag. 183.) *and as it deſerves a Reply, ſo he beſtows an Anſwer of Two whole Pages upon it. He returns,* That a Man that urges this, muſt ſuppoſe Levity and Gravity not to be meer Relations; the Compariſons of certain Situations and external Preſſures: But that they are real Beings, and abſolute inherent Qualities, which is now by every Body exploded, and contrary to the Rules of *Mechanicks*. *Here now is the Solutions: From whence he diverges immediately as far as the original* Chaos *it ſelf, where he ſays,* That there cou'd be no Gravity or Levity; *and,* That theſe Qualities wholly depend on the Conſtitution and Fabrick of the Univerſe. *As for the* Chaos *it ſelf, he affirms,* The Notion of it is as confuſed and monſtrous, as the Import of the Name; and built in every Step on Suppoſitions that are not only arbitrary, but utterly falſe and chimerical. *Not that he* *wou'd be thought to affront* Moſes, *or derogate any thing from the* Scripture, *which ſpeaks of a* Chaos *expreſly; but only coming thus far, to be at the greater Diſtance from a* Vacuum; *he added this by way of Elucidation and Enlargement. Now, had he had any real Spite at the Argument, he wou'd not have gone to telling a Tale of this, and that, and the other Thing that a Man muſt ſuppoſe that urged it; for that's meer Trifling: But preſently endeavour'd to have ſhewn, how this Fact, which is plain and undeniable,* viz. *That Bodies of equal Magnitude have different Specifick Gravities; how this, I ſay, either had not been ſufficient to infer a* Vacuity; *or elſe how his Doctrine of the* Eſſential Activity of Matter *might notwithſtanding have been maintain'd. This had been fair and cloſe to the Purpoſe, whereas what he does is very remote and far from it. 'Tis not a Jot to the Matter in Hand (as to the main Reſult) what Gravity is, or what it depends upon; if it be true and certain in Nature and in Fact, that Bodies of equal Bulk are thus unequally heavy. For this will manifeſtly infer, that the Quantity of Matter in the One cannot be ſo great as that in the Other; neither is the Conſequence to be avoided, but by running away from't, as he does.*

Again; *As to the old Argument for Vacuity, that without this a Body mov'd by another wou'd find no Place nor Room to betake it ſelf into. To this he anſwers (with his wonted Moderation towards all Objections* ;) For you, (*ſays he to the Opponent*, Pag. 178.) to ſpeak in this Manner, is not only to have the ſame groſs Conceptions with your Farmers, but alſo to ſuppoſe the Points *B* and *C*, and all, or moſt of the Points about them, to be really fix'd and in abſolute Repoſe. *But then the main Spirit of the Anſwer is behind*; *for he tells him, That if he ſucceeds in proving the* natural, eſſential, intrinſick *and* neceſſary *Motion of Matter, then all thoſe Difficulties will vaniſh, and appear to be nothing. Which is as much as to ſay, He'll let the Argument alone for this time, but the next time he takes it in hand he intends it ſhall not come off upon ſo eaſie Terms. So that he fairly diſcharges it for the preſent, and bids it expect the Reſult of his having prov'd Motion eſſential to Matter. And perhaps this is more Civility than cou'd have been expected from any Man but Mr.* Toland. *Another Perſon that had confronted an Objection in this Manner wou'd hardly have parted with it without making one Attack at leaſt* ; *whereas he contents himſelf with beſtowing a ſmall Threatning upon it, and ſo lets it go without any farther Harm.*

But farther yet: 'Tis worth taking Notice how little Deſign to ſatisfie, there appears, in his Method of illuſtrating Difficulties, as well as anſwering Objections. Now his Way of doing this is, to make the very Expedient that is to remove one Difficulty, involve Two or Three more that are equally puzzeling. For Inſtance (Pag. 176.) *he tells us,* That to believe the Activity of Matter will clear up all dark and doubtful Points about *Avacuum.* For (*ſays he*) as thoſe particular or limited Quantities which we call ſuch or ſuch Bodies, are but ſeveral Modifications of the general Extenſion of Matter in which they are all contain'd, and which they neither increaſe nor diminiſh; ſo all the particular Local Motions of Matter are but the ſeveral Determinations of its general Action, directing it this or that Way, by theſe or thoſe Cauſes, in this or that Manner, without giving it any Augmentation or Diminution. *Now were Mr.* Toland *very ſerious, an intelligent Reader wou'd think he deſign'd to affront his Underſtanding by offering ſuch a Thing as this to him for the Solution of a Difficulty. And who cou'd do leſs, to ſee a Man cunningly top a Notion upon him (while he pretends to inform him) that is ſo abſolutely precarious and obſcure, as his Notion of particular Bodies and their Motions?*

tions? They are the Modifications of the general Extenſion, and the general Action of Matter. *But how is this made out? Why Mr.* Toland *ſays 'tis ſo: But if Mr.*Toland *ſhou'd ſwear 'tis ſo, and curſe all Mankind into the Bargain that won't believe him, 'twill never paſs for a Truth, for all that, amongſt People that are guided by Reaſon. Ought not he to have put it paſt Diſpute, that particular Bodies were no more than ſo many Modifications of the general Extenſion of the Univerſe, before he had aſſum'd it to ſolve a Difficulty withal? Or if he had done it any where afterwards, it wou'd have been ſufficient. But never to do it, and yet to ſuppoſe it, is to beg his Point in the moſt ſordid Manner imaginable, and to trifle with his Reader unpardonably, if he were not in Jeſt.*

*I know well enough that the Notion of Infinite Matter is a Darling with him, and that he may poſſibly truſt to this, for the Proof of this Definition of particular Bodies. But I alſo know well enough that the Notion of Infinite Matter may be infinitely falſe and abſurd, for any thing he has offer'd to prove the Contrary. And whereas he ſeems to be trying at it (*Pag. 213, &c.*) 'tis evident to any one (that can ſee when a Man preſſes on an Argument with Courage, and when he does it timorouſly) by the Doubtfulneſs of his Proceeding,*

*ceeding, how much he ſuſpected the Infirmity of his Reaſoning upon that Head. However, there needs no more to be ſaid; for (*Page 215.) *he ſets the Infiniteneſs of Matter upon the ſame Foot with the Eſſential Activity of it, and he before took Care to offer no more than a Paralogiſm for the Proof of that Doctrine.*

In the next Place; What Performances he gives his Reader Encouragement to expect from him, may be ſeen at Pag. 159. *where he ſpeaks of no leſs Things, than accounting for the ſame Quantity of Matter in the Univerſe, proving that there can be no* Vacuum, *ſolving all the Difficulties about the* Vis Motrix, *with a great many more beſides theſe. And all this from the Principle of the Eſſential Activity of Matter: Nay more than that, he affirms that this Principle alone can do it. But if any impartial Reader can find, that he has either evinced the Eſſential Activity of Matter; or that he has fairly accounted for theſe Difficulties, or any One of the whole Number; or that he has (much more) ſhewn that no other Principle can do it, but that alone; I'll then beg his Pardon for aſſerting that he has not. In the mean time, I can't tranſcribe his whole Book, but every Reader that will take the Pains may ſee whether I am in the Right or no.*

Laſtly; He makes no Scruple of ſetting any of his own unprov'd precarious Opinions, in Competition with Things that have convincing Proof and Evidence of their ſide, and are univerſally own'd to be ſuch. 'Tis a Point that Mr. Newton *has ſet paſt all Diſpute, that the Weights or Gravitating Forces of Bodies at equal Diſtances from the Center, are proportional to the Quantity of Matter in them. But with Mr.* Toland *this is yet a Controverſie* ---- Leaſt I engage my ſelf in a Diſpute (*ſays he*) about the Nature of Gravity, as whether the Weight of Bodies be proportional to the Quantity of Matter, *&c.* Pag. 207.

Again; 'Tis a Truth ſufficiently clear, that the external Preſſures of the Atmoſtphere *and* Æther, *have nothing to do, as cauſes either in whole or in part of ſuch an Effect as Gravity in Bodies; therefore external Preſſures can be of no Significancy to the Producing ſuch an Effect as the different Specifick Gravities of Bodies. But he is pleaſed to declare his Opinion to be otherwiſe, and allow Preſſure a Share in that* Phenomenon.------ Tho' their Specifick Gravities (*ſpeaking of Cork, Lead, Gold, and the like*) be ſo different from each other; proceeding partly from *external Preſſures,* and partly from thoſe internal Structures and Modifications, which give their common Matter thoſe various

Forms that conſtitute their Species, *&c.* *Ibid.* This *(ſays he)* is my own Opinion, whatever be my Reaſons for it. *Which is admirably well ſaid; and perhaps he knew well enough why he ſhou'd keep his Reaſons to himſelf. But however; this very Paſſage alone is enough to juſtifie my Surmiſe, that Mr.* Toland *wou'd be thought to banter. For to offer an Opinion that thwarts a Matter of Demonſtration, and ſay, I believe thus, and thus whatever be my Reaſons for't, is almoſt the ſame Thing as to ſay,* I am in Jeſt.

In ſhort, Mr. Toland *appears to me, to manage the Point much after the ſame Manner with* Lord Peter *in the* Tale of a Tub. *The One calls a brown Cruſt, Mutton and Claret; as the Other does Fancy and Paralogiſm, Reaſon and ſound Philoſophy. Both appear grave and ſerious in what they do and ſay, and agree in acting an impoſing Part; One upon the Sences, the Other upon the Underſtandings of thoſe they talk to. The Difference is, that One ſwears to the Truth of what he ſays, and the Other is ſo modeſt as to pawn his Word for't. Tho' there's no queſtion but* Peter *might have ſome Reaſons too lying by him, to prove what he ſaid to be true: Only (like this Gentleman) he was not pleaſed to produce them. 'Twas ſufficient for him to ſay, 'Tis my Reſolution, that this* *which looks like a brown Cruſt, is really what I have told you*, whatever be my Reaſons for't. *But* Peter'*s Mutton and Claret was certainly never more like a Piece of a brown Loaf, than Mr.* Toland'*s Philoſophy is like Paralogiſm. The Reſemblance is ſuch, that I doubt not, he has or will find, 'tis univerſally taken for the very ſame Thing in the World.*

I ſhall ſay no more; but only that I hear, ſome Queſtions *will ſhortly be propoſed to him upon the Buſineſs of* Real Space. *For my own Part I have nothing to do with that Point, and as much as I value the Judgments of ſome that defend it, in other Matters, cannot ſubſcribe to them in this. I do believe, as well as Mr.* Toland, *that many of thoſe Gentlemen firmly believ'd the Exiſtence of a Deity. But I alſo believe too, that* Infinite neceſſarily-ſelf-moving Matter *may ſerve to entertain an Atheiſt, as well as* Almighty Space.

It remains only now, for me to return my humble Thanks to the Reader, and bid him, Farewell.

N. B. The Reader may, if he pleaſes, omit the *Lemma* (at *Pag.* 138.) and take the whole Proceſs (for the Inveſtigation of the following *Theor.* 4.) thus. Uſing all the Symbols as in the *Lemma*, from the Nature of the Circle 'tis $YY = 2RX - XX$, and $yy = 2rx - xx$; alſo $D^2 = 2RX$, and $d^2 = 2rx$. But at laſt D and d are coincident with A and a, and X, x with $\dot{X}$, $\dot{x}$; ſo that then we have $A^2 = 2R\dot{X}$, and $a^2 = 2r\dot{x}$, therefore $\dot{X} : \dot{x} :: \frac{A^2}{2R} : \frac{a^2}{2r}$. Q. E. I. In this manner, I ſay, the Reader may come to the *Theorem*, without the Aſſiſtance of the *Lemma*, which gives the laſt *Ratio* of the Subtenſe of the Angle of Contact to the conterminal Arch. Tho' the *Lemma* it ſelf too be contain'd in the Proceſs; for ſince $A^2 = 2R\dot{X}$, and $a^2 = 2r\dot{x}$, therefore if $2R = 2r$, then $\dot{X} : \dot{x} :: A^2 : a^2$; as Directions are given for the Reading of it in the *Errata*.

Fig. 19.

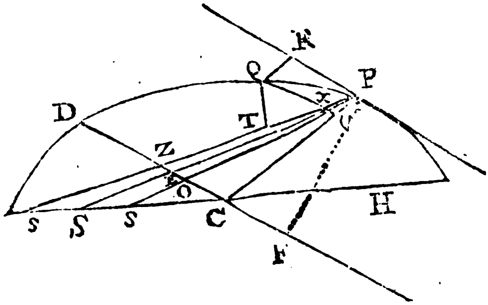
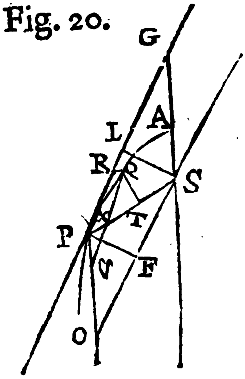


Fig. 20.



OF THE LAWS of MOTION, WITH THEIR Application to *Mechanicks*.

PART I.

LAW I.

ALL Bodies will perſevere in their State of Reſt, or uniform direct Motion, unleſs they are compell'd to change that State by ſome Force impreſs'd upon them.

LAW II.

The Alteration of Motion is ever proportional to the impreſs'd Force (that cauſes it,) and the Direction of it is in the ſame Right Line with that, according to which the Force is impreſs'd.

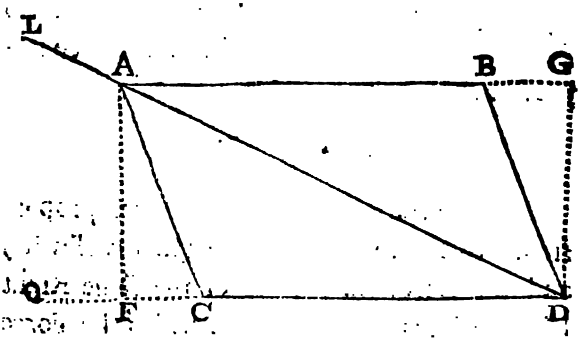
LAW III.

Reaction is ever equal and contrary to Action; or the Actions of Two Bodies upon one another are always equal, and have contrary Directions.

From theſe Laws the Illuſtrious Author deduces the following Corollaries.

Firſt COROLLARY *to the Laws of Motion.*

A Body will deſcribe the Diagonal of a Parallelogram by Two ſeveral Forces combin'd, in the ſame Time that it would deſcribe the one or the other of thoſe Sides by the one or the other of thoſe Forces ſeparately conſider'd.



DE-

DEMONSTRATION.

Suppoſe a Body cou'd deſcribe the Line AB (*FIG.* I.) in the Time T, with the uniform Motion M; and in the ſame Time, the Line AC, with the uniform Motion N. Compleat the Parallelogram, and draw the Diagonal AD. The Diagonal AD ſhall be deſcribed by theſe Two Forces thus combin'd in the ſame Time T: For the Force N acting with the Direction AC, parallel to BD, cannot change the Velocity of the Motion towards BD; the Body therefore will be carry'd to the Line BD in the ſame Time T, whether the Force N be impreſs'd or no; therefore at the End of the Time T it will be found ſomewhere in the Line BD. In like manner, the Force M acting with the Direction AB, parallel to CD, cannot change the Velocity of the Motion towards CD: Therefore the Body will be carry'd to the Line CD, in the ſame Time T, whether the Force M be impreſs'd or no; conſequently at the End of the Time T it will be found ſomewhere in the Line CD. Therefore ſince at the End of that Time it will be found in both theſe Lines BD and CD, it will certainly be found in D their Interſection, that is in the Diagonal Q. E. D.

From hence ariſe the following Theorems.

THEOR. I.

The Forces by which the Sides and Diagonal of a Parallelogram are deſcribed, are proportional to, and may conſequently be expounded by thoſe Sides, and that Diagonal reſpectively. For the Motions being uniform by the Hypotheſis, and the Times of the Deſcription (both of the Sides and Diagonal) being prov'd to be the ſame by the foregoing Demonſtration, the Spaces deſcribed ſhall be as the Velocities by *Law* II. or as the Forces to which theſe Velocities are proportional reſpectively. Therefore (putting the Force in AD=G) it will be G. *M* : : AD. AB and G. N : : AD. AC and G. M+N : : AD. AB+AC.

THEOR. II.

If a Body with a Force proportional to AD deſcribes the Right Line AD, the Motion, and Direction of that Motion will be the ſame as if the Body had at firſt been impell'd by Two Forces, acting according to the Directions AB and AC, and proportional to thoſe Lines. Therefore any Motion, tho' in it ſelf ever ſo ſimple, may be conſidered as compounded of others, and (from this Principle) may be reſolved into them.

THEOR. III.

Suppoſe the Parallelogram as before, and a Body to be held immovable in the Place A by Two equal Forces acting with contrary Directions; the one tending from A to D, and the other from A to L. I ſay 'tis the ſame thing as if it were held by Three Forces acting with the Directions from A to L, from A to B, and from A to C. For the Force AD (by *Theor.* II.) is equivalent to the Forces AB and AC. Therefore, if the Body be held immovable by the Forces, whoſe Directions are AD and AL, it will be held by the Forces acting with the Directions AL, AB and AC.

THEOR. IV.

A Body being thus held immovable by Three Powers; thoſe Powers are one to another directly as the Lines that are drawn parallel to their reſpective Directions, and terminated at the Point of their mutual Concourſe. For (by *Theor.* I.) the Powers acting with the Directions AD, AB, AC are in Proportion as thoſe Lines AD, AB, AC reſpectively taken; that is, as the Lines parallel to the Directions, and terminated at their common Interſection.

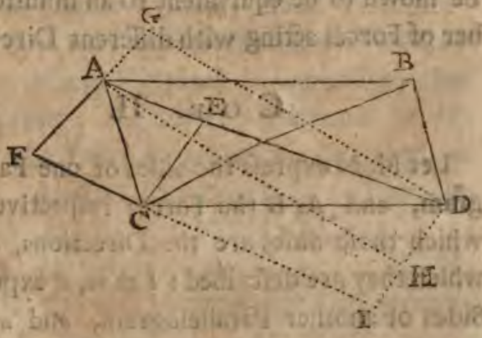
THEOR. V.

Therefore any Two of theſe Powers are to one another reciprocally as the Sines of the Angles which their Lines of Direction make with the Line of Direction of the Third Power. Thus the Power M (whoſe Direction is AB) is to Power N (whoſe Direction is AC) as AB to AC or BD, that is (becauſe in any Triangle the Sides are proportional to the Sines of the oppoſite Angles) as the Sine of the Angle ADB or CAD, to the Sine of the Angle DAB. Now CAD is the Angle which the Line of Direction of the Power N makes with AD the Line of Direction of the Third Power; and DAB is the Angle which the Line of Direction of the Power M makes with the ſame Line AD: Therefore the Powers are as the Sines of thoſe Angles reciprocally. Thus alſo the Power G (whoſe Direction is AD) is to the Power M; as the Sine of the Angle BAC, is to the Sine of the Angle DAC; that is, as the Sine of the Angle ACQ, or the Sine of the Angle ACD (the Complement of the Former to the Two Right Angles) is to the Sine of the Angle DAC; that is, as AD to CD.

General COROLLARIES.

COR. I.

A Force acting with any given Direction may be ſhewn to be equivalent to an infinite Number of Forces acting with different Directions. For one and the ſame Line may be the Diagonal of an infinite Number of Parallelograms. Thus the Force in AD in the Parallelogram (*FIG.* II.) ABCD is equivalent to the Forces in AB and AC; and in the Parallelogram AGDH the ſame Force is equivalent to the Forces in AG and GD, and ſo of infinite others. Alſo ſince the Sides of the Parallelogram ABCD may be made the Diagonals of an infinite Number of others, the Force in CB may be ſhown to be equivalent to an infinite Number of other Forces. For Example; Make the Side AC the Diagonal of the Parallelogram FAEC, and then



the Force acting according to the Direction CB will be equivalent to the Forces acting according to the Directions CF, CE and CD; and if CD be made the Diagonal of the Parallelogram CEDI, the ſame Force in CB will be equivalent to the Forces acting in CF, CI, and double the Force according to the Direction CE, and ſo continuing to make the Sides of theſe laſt Parallelograms the Diagonals of others, and ſo on infinitely, the Force in CB may be ſhown to be equivalent to an infinite Number of Forces acting with different Directions.

C o r. II.

Let M, N expreſs the Sides of one Parallelogram, and A, B the Forces reſpectively, of which thoſe Sides are the Directions, and by which they are deſcribed: Let m, n expreſs the Sides of another Parallelogram, and a, b the Forces. Alſo let the Diagonal common to both be D, and the Force whoſe Direction is in that Line be F. Then (by *Theor.* I.) $A+B.\ F :: M+N.\ D$, and $a+b.\ F :: m+n.\ D$. Therefore $A+B : a+b :: M+N : m+n$. That is, the Sums of the Forces in the Sides of the Two Parallelograms, are directly as the Sums of the Sides.—Likewiſe (from the forementioned *Theorem*, becauſe $A : F :: M : D$, and $B : F :: N : D$. and conſequently $A \times D = F \times M$, and $B \times D = F \times N$;

therefore $A\times D - B\times D = F\times M - F\times N$; that is $A-B : F :: M-N : D$. And ſo for the other Parallelogram, it may be ſhewn, that $a-b : F :: m-n : D$. Therefore (by Equality of Proportion) $A-B : a-b :: M-N : m-n$. That is, the Differences of the Forces in the Sides of the Two Parallelograms, are directly as the Differences of the Sides.

COR. III.

Let the former Symbols ſtand as before; only let m, n, now expreſs the Sides of a Parallelogram, whoſe Diagonal is unequal to that of the former; and let this Diagonal be put $=d$, and the Force by which 'tis deſcribed be put $=f$. Alſo let it be $D : d :: F : f$. Then (by *Theor.* I.) $A+B : F :: M+N : D$, and $a+b : f :: m+n : d$; therefore $A+B : a+b :: M+N : m+n$. It may alſo be proved in like manner, as in *Cor.* II. that $A-B : a-b :: M-N : m-n$. So that tho' the Parallelograms have unequal Diagonals, yet thoſe Diagonals being deſcribed by uniform Forces proportional to the Lengths of them; the Sums or Differences of the Forces, that deſcribe the Sides of thoſe Parallelograms, ſhall be directly proportional to the Sums or Differences of the Sides themſelves.

COR. IV.

In every Rectangular Parallelogram, the Force in the Diagonal is equal in Power to the Forces in the Sides. This is *Theor.* II. of *Galilæus*'s 4th *Mechanick Dialogue.*

The following Problems relating to this Matter may here not improperly be ſubjoin'd.

PROB. I.

To find Parallelograms having a common Diagonal, in which the Sums or Differences of the Forces (by which the Sides are deſcrib'd) ſhall ſtill be equal to one another, or be expreſs'd by a ſtanding Quantity.

According to the Tenour of *Cor.* II. let the Quantity $M+N$ be the Tranſverſe Ax of any Ellipſe, and D be equal to the double Excentricity, or to the Diſtance of the Foci from one another. Alſo let M, N be Two Lines drawn from the Foci to any one and the ſame Point in the Curve, and m, n Lines drawn from the Foci to any other Point in the Curve. I ſay, that the Lines M, N, and m, n, are the Sides, and that D is the common Diagonal of Two ſuch Parallelograms as are required. For from the Nature of the Ellipſe, $M+N$ is $=m+n$ perpetually; and therefore $A+B=a+b$ ever alſo, by *Cor.* II. Q. E. I.

If the Differences of the Forces were required to be a ſtanding Quantity, and ſuch Parallelograms were to be determin'd. Then let $M-N=$ the Tranſverſe Ax of any Hyperbola, and $D=$ the Diſtance of the Foci. Then from the Nature of this Curve, $M-N$ ſhall $=m-n$, and conſequently $A-B=a-b$ perpetually.

PROB. II.

To find Parallelograms having unequal Diagonals, in which the Sums or Differences of the Forces, with which the Sides are deſcribed, ſhall ſtill be equal, or be expreſs'd by a ſtanding Quantity.

According to the Tenour of *Coroll.* III. let $M+N$ be the common Tranſverſe Ax of any Two Ellipſes, in which D and d are the Diſtances of the Foci; D for one Ellipſe, and d for the other. Then if M, N, and m, n expreſs Lines drawn from the Foci to any Points in the Two Curves (that is, M, N from the Foci of the one Ellipſe to any Point in that Curve, and m, u from the Foci of the other Ellipſe to any Point in that Curve) I ſay, that theſe are the Sides, and the Focal Diſtances the Diagonals of ſuch Parallelograms as are required. For ſince $M+N$ is $=m\times n$ perpetually (from the Nature of the Ellipſe) therefore alſo, according to the Intent of that 3d *Cor.* $A+B$ ſhall be ever

=

$=a+b$. Q: E: I. So if $M-N$ were the common Tranſverſe of Two Hyperbola's, in which the Diſtances of the Foci were D and d, then M, N, and m, n being Lines from the Foci, as before; the Parallelograms will be ſuch as is required: And $M-N$ being ever $=m-n$ from the Genius of the Hyperbola, $A-B$ ſhall $=a-b$; that is, the Differences of the Forces ſhall be always equal.

PROB. III.

To find Parallelograms having either equal or unequal Diagonals, in which the Sums or Differences of the Forces with which the Sides are deſcrib'd, ſhall ſtill be in any *Ratio* aſſign'd.

Let the *Ratio* aſſign'd be that of $p. q$.

CASE I.

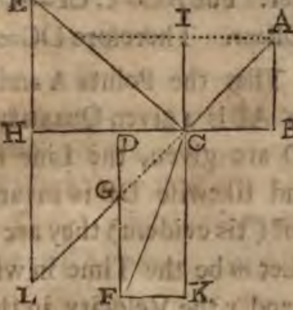
Let the Parallelograms be requir'd to have equal Diagonals. Put the Tranſverſe Ax of any Ellipſe or Hyperbola $=T$, the Parameter $=L$, the Tranſverſe Ax of any other Ellipſe or Hyperbola $=t$, and the Parameter $=l$. 'Tis manifeſt, that in order to the ſatisfying the Demands of the Problem, the Ellipſes or Hyperbola's muſt be ſo proportion'd to each other, that their Tranſverſe Axes may be in the *Ratio* aſſign'd, and their Focal Diſtances equal. One Ellipſe or Hyperbola therefore being given, whoſe

Tranſverſe and Parameter, T and L, are known: I ſay, that another Ellipſe whoſe Tranſverſe Ax t is $= \frac{Tq}{p}$, and its Parameter $l = \frac{Tq^2 - Tp^2 + Lp^2}{qp}$; or another Hyperbola, whoſe Tranſverſe $= \frac{Tq}{p}$, and its Parameter $= \frac{Tp^2 + Lp^2 - Tq^2}{qp}$; theſe 2 Ellipſes, or theſe 2 Hyperbola's ſhall have the Conditions required, and the Parallelograms (whoſe Diagonals are the Diſtances of the Foci, and whoſe Sides are the Lines drawn from the Foci to any Points in the Curves) ſhall be ſuch as will anſwer the Demands of the Problem. The Demonſtration of which is eaſie: For ſince the Square of the Diſtance of the Focus from the Center, is in the Ellipſe = to the Difference, and in the Hyperbola = to the Sum of the Squares of the Semitranſverſe and Semiconjugate; therefore in the Two Ellipſes or Hyperbola's, if the Diſtances of the Focus from the Center in one be put equal to that in another, we have this Equation $T^2 - LT = t^2 - lt$ for the Ellipſes, and $T^2 + LT = t^2 + lt$ for the Hyperbola's. From whence L and T being given in the one, and alſo t in the other, the Parameter l of that other may be found, and the Value of it will be ſuch as is determined above. For becauſe t is $= \frac{Tq}{p}$, and conſequently Given; therefore the Value of *l* will be expreſs'd all in the Terms of the Given Quantities L, T, *p*, *q*. Laſtly; Becauſe the Tranſverſe Axes are in the Given *Ratio* of *p* to *q*, by the Hypotheſis; and the Sums or Differences of the Lines from the Foci to the Curves are equal to the Tranſverſe Axes from the Nature of theſe Sections, and therefore ever in the ſame *Ratio* of *p* : *q*; and becauſe (by *Cor.* II.) the Sums or Differences of the Forces are ever proportional to the Sums or Differences of the Sides (which are the Lines drawn from the Foci to the Curves:) Therefore the Sums or Differerces of the Forces ſhall always be in the Given *Ratio* of *p* : *q*. Q: E: D.

CASE II.

Let the Diagonals be required to be unequal. If T be the Tranſverſe of one Ellipſe or Hyperbola, then $\frac{Tq}{p}$ ſhall be that of the other. And if the Diſtance of the Foci in one Ellipſe or Hyperbola be D, and that in the other be *d*; theſe being the Diagonals, and the Lines drawn from the Foci to the Curves being the Sides of the Parallelograms: If the Diagonals be deſcrib'd by Forces proportional to their Lengths, the Porallelograms thus made ſhall be ſuch as are required. The 3d *Corollary* (with thoſe ſo often mention'd Properties of the *Conick Sections*) demonſtrate this without any further Trouble.

PROB.



PROB. IV.

The Points A, B, F, D, (*FIG.* III.) being given, 'tis required to determine the Point C in the Right Line HB (given in Poſition ;) ſo that compleating the Rectangular Parallelograms ABCI, DCFK, the Time of deſcribing the Oblique Path ACF (which is the Sum of the Diagonals) may be the leaſt poſſible ; or (we may propoſe it thus) that the Sum of the Forces with which the Diagonals AC and CF are deſcribed, may be the leaſt poſſible.

Here I ſuppoſe the Sides IC and CK to lie in a Right Line : Alſo that the Body, by ſome Cauſe or other, does at the Point C receive another, and different Direction of its Motion, and a Change of its Volocity. In a Word, that whereas before it came to the Point C, it mov'd in the Diagonal AC, it afterwards moves in the

Diagonal CF, making an Angle as GCF with the former. Put AC$=z$. CF$=v$. BC$=x$. AB$=d$. FD$=b$. DB$=c$. Therefore DC$=c-x$.

Note, That the Points A and B are given; therefore AB is a given Quantity. So becauſe F and D are given, the Line FD is ever the ſame; and likewiſe DB is invariable. As for all the reſt ('tis evident) they are flowing Quantities. Let m be the Time in which AC is deſcribed, and r the Velocity in the ſame, n the Time in which CF is deſcribed, and s the Velocity; which Velocities I ſuppoſe to be the ſame in all the Points of the Lines AC and CF reſpectively, and conſequently the Quantities r and s to be ſtanding ones. From the common Principles of Mechanicks, 'tis evident, that $m.n :: z\times s. r\times v$. That is, the Times are in the *Ratio* compounded of the direct *Ratio* of the Spaces, and reciprocal *Ratio* of the Velocities. Alſo, if m, n, expreſs'd Forces, and r, s, Reſiſtances, then the Forces wou'd be in the *Ratio* compounded of the direct *Ratio* of the Spaces, and the direct *Ratio* of the Reſiſtances. Therefore if s denotes the Reſiſtance in AC or z, and r that in CF or v; then one and the ſame Expreſſion ſhall ſerve with Reſpect to both theſe Ways of propoſing the Problem. Therefore (in one Suppoſition) the Time in which the whole Space ACF is deſcribed (or the Sum of the Forces in the other) will be as $zs+rv$.

But from the Rectangular Triangles we have $z=\overline{xx+dd}|^{\frac{1}{2}}$ and $v=\overline{bb+cc-2cx+xx}|^{\frac{1}{2}}$; Therefore the Expreſſion comes to this $s+\overline{xx\times dd}|^{\frac{1}{2}}$ $+r\times\overline{bb+cc-2cx+xx}|^{\frac{1}{2}}$; which Expreſſion is to be determin'd to a *Minimum.* Therefore working by the admirable Method of Fluxions, we ſhall find

$$\frac{s\times x\dot{x}}{\overline{xx+dd}|^{\frac{1}{2}}}+\frac{r\times x\dot{x}-c\dot{x}}{\overline{bb+cc-2cx+xx}|^{\frac{1}{2}}}=0$$

than is, $\frac{s\times x}{z}=\frac{r\times \overline{c-x}}{v}$. From whence $x.\overline{c-x}::$ $r\times z,\ s\times v.$ Therefore if the Points A and F be ſuppoſed to lie in the Circumference of a Circle of which C is the Center; that is, if AC=CF, then it will be $x.\overline{c-x}::r.s.$ And conſequently if the Line DB be divided in C, ſo that BC : DC, as the Velocities in the Diagonals AC, CF directly, or as the Reſiſtances reciprocally; that is, if the Sines of the Inclinations of the Diagonals AC, CF to the perpendicular ICK, are in either of theſe *Ratio's*, then the inflex'd Path ACF ſhall be deſcribed in the leaſt Time, of all that are in like manner comprehended between any Two Lines drawn from the Points A, F, to any Point in the Line DB, or (with Reſpect to the other Way of propoſing the Problem) the Sum of the Forces with which the Two Diagonals AC, CF are deſcribed, ſhall be the leaſt poſſible. Q. E. I.

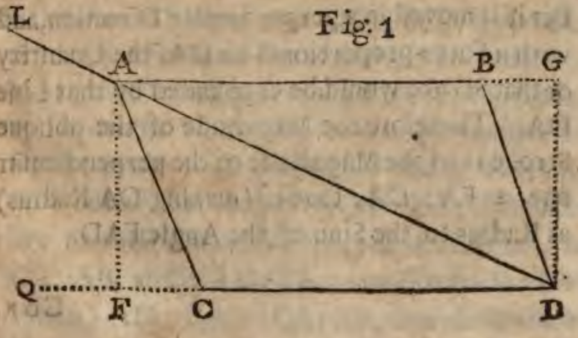
To carry this on a little farther; If $r=s$, that is, if (the Body meeting with nothing at C to retard its Motion) the Velocities in AC and CF be the ſame; then $x.\ c-x :: z.\ v.$ In which Caſe FC will be coincident with LC, which makes one Right Line with AC. Alſo LH being perpendicular to BD produced, the Point D becomes now coincident with H. The Triangles ABC, LCH therefore are ſimilar. For BC being $=x$ (as before) CH is now $c-x$, and BH$=c$. But 'tis found, that $x.\ c-x :: z.\ v.$ that is, BC. CH :: AC. CL, and the Angles ACB and LCH are equal; therefore, *&c.* From hence making HE = LH, the Triangle HEC will be equal and ſimilar to HLC, and alſo ſimilar to ACB. Now if the Body (moving in the Direction AC) inſtead of paſſing on in the Right Line CL, were reflected at the Point C, then ſince the Velocity continues ſtill the ſame (the Direction only being changed) in the ſame time that it would deſcribe CL, if it went on directly, it will deſcribe a Right Line =to CL in its reflex'd Motion. But from the Equality and Similarity of the Triangles HLC and HEC the Line CE is = CL, therefore it will deſcribe CE. But the Angle HCE is = (HCL=) ACB; therefore the Body will be reflected ſo, that the Angles of Incidence and Reflection (ACI and ECI) ſhall be equal.

N. B. I abſtract here from all the Imperfections of Matter, and ſuppoſe the Body perfectly Elaſtick, and the Obex firm.

SCHOL.

If AC were ſuppoſed to be a Ray of Light, the Theorems found would be thoſe Two celebrated ones in *Dioptricks* and *Catoptricks*, upon which thoſe Sciences are built. And DC, CB, which are the Sines of the Inclinations of the Diagonals FC, AC, to the perpendicular ICK; would be the Sine of the refracted Angle, and the Sine of the Angle of Incidence, if DCB were the Surface of a different *Medium*. From whence thoſe Sines would be as the Velocities in the Two *Mediums*, by what was found above, and as Mr. *Newton* has ſhown *Prop.* 95. *Lib.* 2. But this only in *Tranſitu*. The general Principle concerniug the Compoſition of Forces ſhall now be apply'd to ſeveral Problems in *Mechanicks*.

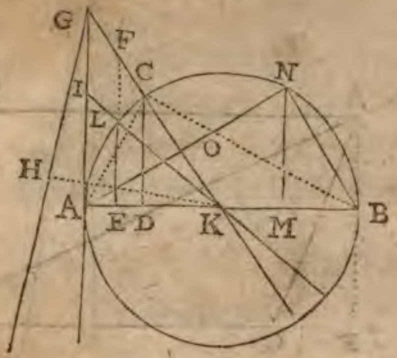
Fig. 1



PROB. I.

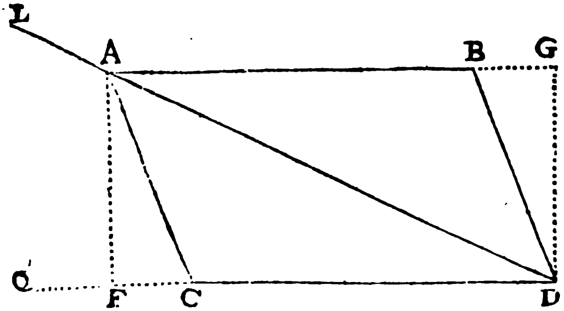
To aſſign the Proportion of a Stroke made with any oblique Direction upon a Plane, to one made with a perpendicular Direction, the Body moving in both Directions with the ſame Degree of Velocity.

SUppoſe the Body's Direction (*FIG.* I.) to be in the Right Line AD, and that it ſtruck upon the Plane DF at the Point D. Let fall AF perpendicular to FD, and compleat the Parallelogram FDAG. The Motion of the Body in the Line DA is equivalent to Two other Motions whoſe Directions are AF and AG, by *Theor.* II. But the Motion whoſe Direction is AG is of no Significancy as to the Stroke upon the Plane DF, for GA being parallel to DF, the Body moving with that Direction would never meet the Plane. Therefore the Force by which the Body moves in AD, being as AD; that by which it ſtrikes upon the Plane, is as AF. But if it moved in a perpendicular Direction, and with a Force proportional to DA, the Quantity of that Stroke would be expounded by that Line DA. Therefore the Magnitude of the oblique Stroke is to the Magnitude of the perpendicular one, as FA : DA, that is (making DA Radius) as Radius to the Sine of the Angle FAD.



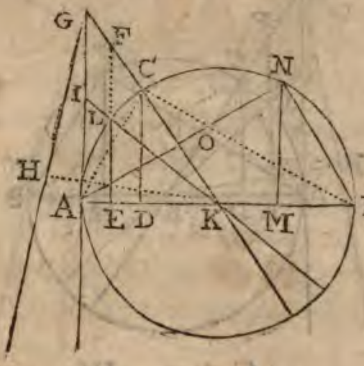
Cor. I.

The Angles of Obliquity upon the Planes being equal, the oblique Strokes are directly proportional to the perpendicular ones, how unequal ſoever the Velocities (with which the perpendicular Strokes are made) be. Thus in the Rightangled Triangle ACB (*FIG.* IV.) the perpendicular CD being let fall; ſince ABC = ACD, therefore the oblique Stroke with the Direction AB upon the Plane BC, is to the perpendicular Stroke made with the ſame Velocity, as the oblique Stroke with the Direction AC upon the Plane CD, to the perpendicular Stroke made with the ſame Velocity with this oblique one. For (by the *Prop.*) the oblique Strokes are to their reſpective perpendicular ones, as CA : AB, and as AD : CA; but (from the Elements) AD : CA :: CA : AB, therefore the oblique and perpendicular Strokes are directly proportional.



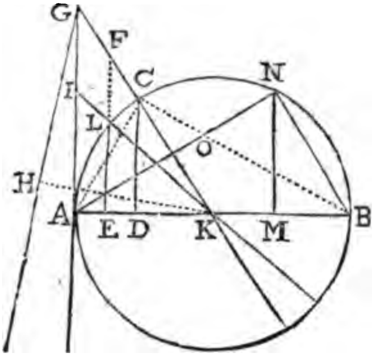
C O R. II.

The oblique Stroke made upon the Sides of any Rectangle, with equal Velocities, and with the Direction of the Diagonal, do anſwer reciprocally to the Forces that deſcribe thoſe Sides, and by the Compoſition of which the Diagonal is deſcrib'd. For (*FIG.* I.) the Quantity of the perpendicular Stroke upon the Sides AF, AG, being expounded by the Diagonal DA, the oblique ones made upon the ſame Sides with the ſame Velocity, and the Direction DA, ſhall be as FD and DG. The Force by which the Diagonal is deſcribed being expounded by DA, the Forces in the Sides AF, AG ſhall be as DG and FD. So that the Magnitudes of the Strokes, and the Forces do anſwer reciprocally to one another.



Cor. III.

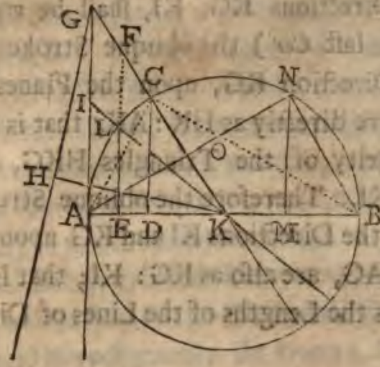
A Body moving with equal Velocities in the Lines of Direction of equal Length, but unequally enclin'd to Two Planes; the Magnitudes of the oblique Strokes ſhall be directly as the perpendicular Altitudes of the Lines of Direction. Thus (*FIG.* IV.) if KC = KL, the Strokes with theſe Directions upon the Planes CD, LE, ſhall be as KD and KE. For the oblique Stroke upon CD, is to the perpendicular one, as KD : KC; and the oblique Stroke upon LE, is to the perpendicular one, as KE : KL. But the perpendicular Strokes (being made with the ſame Velocity) are equal, and KC=KL by the Hypotheſis; therefore the Magnitudes of the oblique Strokes, upon thoſe Planes, are as KD and KE directly, which are the Altitudes of the Lines of Direction.



Cor. IV.

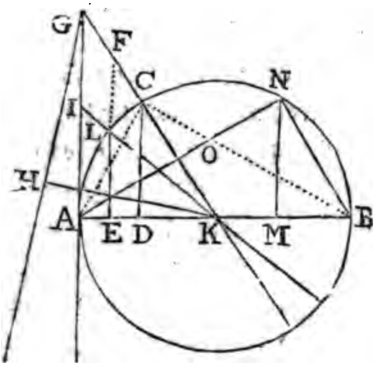
A Body moving with the ſame Velocity in one and the ſame Line of Direction, which is unequally enclin'd to Two Planes (meeting at the ſame Point) the oblique Strokes made upon theſe Two Planes ſhall be directly as the perpendicular Altitudes of the Line of Direction. The Line of Direction being KG, (*FIG.* IV.) let the Line GH be drawn, making the Angle AGH, with the Line AG; alſo let KA, KH be perpendicular to the Lines AG, HG. The Magnitudes of the oblique Strokes made with the Direction KG, upon the Planes AG and HG, ſhall be as KA, KH directly; which is eaſily deduced from what has been ſaid.

Cor.



COR. V.

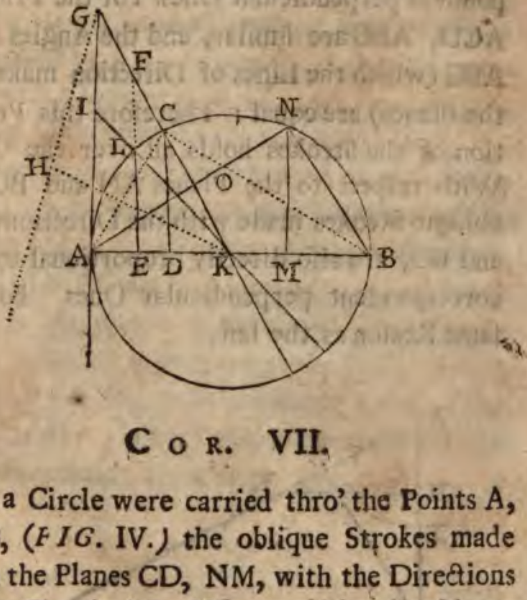
A Body moving with the ſame Velocity in Lines of Direction of unequal Length, and Inclinations to a Plane, but equally high; the Magnitudes of the oblique Strokes are reciprocally, as the Lengths of the Lines of Direction. The Lines of Direction being KG, KI (*FIG.* IV.) let HG be drawn, ſo that the Angle AGH may be=IKG, and let fall the Perpendiculars KA, KH as before. 'Tis evident, that the Angle KGH is = the Angle KIA. Therefore (by *Cor.* I.) ſince the Angles KGH and KIA are equal, the oblique Strokes made with the Directions KG and KI, upon the Planes HG and AI, ſhall be directly proportional to their reſpective perpendicular Strokes. Conſequently, if the Body moves with the ſame Velocity, and ſo the perpendicular Strokes be equal, the oblique ones upon the Planes HG and AI, with the Directions KG, KI, ſhall be equal. But by the laſt *Cor.*) the oblique Strokes made in the Direction KG, upon the Planes GH and GA, are directly as HK : AK, that is (from the Similarity of the Triangles HKG, AKI) as KG : KI. Therefore the oblique Strokes made with the Directions KI and KG upon the ſame Plane AG, are alſo as KG : KI; that is reciprocally as the Lengths of the Lines of Direction.



C O R. VI.

Therefore univerſally (joining the Accounts of *Cor.* III. and V. together) any oblique Strokes made by Lines of Direction neither of equal Lengths, nor equal Altitudes, are in the *Ratio* compounded of the direct *Ratio* of the perpendicular Altitudes, and the reciprocal *Ratio* of the Lengths of the Lines of Direction; *Ex. gr.*

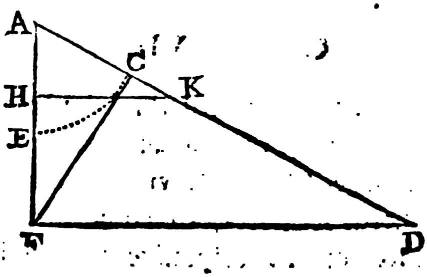
the oblique Stroke upon the Plane CD, with the Direction KC, is to that upon the Plane AI with the Direction KI, as DK×IK : AK×KC. And from hence I infer, that (univerſally ſpeaking) the Magnitudes of any Two oblique Strokes made with the ſame Velocities, are in Proportion directly as the Declivities of the Lines of Direction. For the Declivities of any Two Lines are univerſally as the Altitudes directly, and the Lengths reciprocally. By *Prop.* 14. *Chap.* 2. of Dr. *Wallis*'s *Mechanicks.*



Cor. VII.

If a Circle were carried thro' the Points A, C, B, (*FIG.* IV.) the oblique Strokes made upon the Planes CD, NM, with the Directions of the Chords AC, AN, wou'd be directly as the Squares of the reſpective perpendicular

Strokes upon the ſame Planes. For, with reſpect to the Plane CD, the Magnitude of the oblique Stroke, is to the Magnitude of the Perpendicular, as AD : AC. Likewiſe with reſpect to the Plane NM, the Proportion of the Strokes is as AM : AN. But from the Nature of the Circle, AD : AM :: AC^q : AN^q. Therefore, &c. With reſpect to the Planes CD and BC; the oblique Strokes made upon them, with the Directions AO, AB reſpectively, wou'd be directly proportional to their correſpondent perpendicular Ones. For the Triangles ACD, ABC are ſimilar, and the Angles ACD, ABC (which the Lines of Direction make with the Planes) are equal : Therefore this Proportion of the Strokes holds all over the Circle. With reſpect to the Planes AN and BC, the oblique Strokes made with the Directions AO, and BO, are alſo directly proportional to their correſpondent perpendicular Ones. For the ſame Reaſon as the laſt.



Cor. VIII.

If a Body ſetting out from the Point A (*FIG.* V.) moved in the Directions AF and AD, with an equal Degree of Velocity; the Proportion of the oblique and perpendicular Strokes wou'd be as before; *viz.* as AF : AD.

Let fall the perpendicular FC from the Point F to the Line AD. If the Body moving with a Velocity, as AD, and in a perpendicular Direction, ſtruct the Plane at the Point D, the Magnitude of the direct Stroke at that Point, wou'd be expounded by AD, and that of the oblique Stroke in the Direction AD, wou'd be expounded by AF. Likewiſe, if the Body moving with a Velocity, as AF, and in a perpendicular Direction, ſtruck the Plane at D, the Magnitude of the direct Stroke at that Point would be expounded by AF, and that of the oblique one in the Direction AD, would be expounded by AC (by *Cor.* I.) for AD : AF :: AF : AC. But the Magnitude of the direct Stroke is the ſame, whether it be given at the Point D with a Velocity as AF, or at the Point F with a Velocity as AF. Therefore the Proportion of the Strokes remains the ſame.

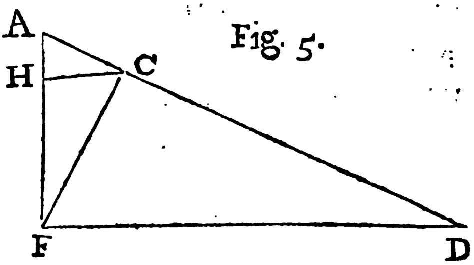


Fig. 5.

Cor. IX.

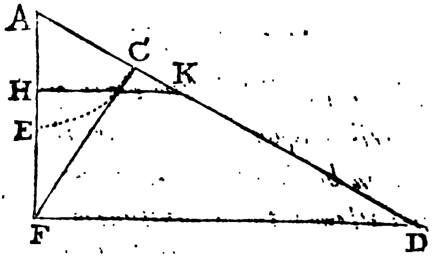
The oblique Stroke (*FIG.* V.) made in the Direction AD, is equal to a direct Stroke (made with a Velocity as much leſs than that which the Body moves in AD; as AF is leſs than AD.)

For the oblique Stroke is to the direct one made with the ſame Velocity as AF: AD. Now any Two direct Strokes made by the ſame Body are one to another, as the Velocities with which the ſtriking Body moves. Therefore if theſe Velocities be as the Lines AF and AD, then the Magnitude of the oblique Stroke with a Velocity as AD, is equal the Magnitude of the direct Stroke with a Velocity as AF: Or, which is all one, the Magnitude of the oblique Stroke with a Velocity as AF, is equal the Magnitude of the direct Stroke with a Velocity as AC.

LEMMA.

If a Body, deſcending by the Force of its Gravity, ſtrikes upon a Plane; let the Line of Direction be either oblique or perpendicular to that Plane; the Magnitude of the Stroke is the ſame, as if the Body had moved all along in that ſame Direction, with a Degree of Velocity, equal to that which is gotten by the accelerated Motion, at the Point where the Line of Direction meets the Plane.

FOR the moving Body and the Line of Direction, being each the ſame, nothing but a different Velocity in ſtriking upon the Plane can make any Alteration in the Magnitude of the Stroke. But whether the Body moves with an equable or with an accelerated Motion, the Velocity with which it ſtrikes upon the Plane is the ſame, if the Velocity of the uniform Motion be equal to that gotten by the accelerated, at the Point where the Line of Direction meets the Plane. Therefore, *&c.* Q: E: D.



THEOR. I.

If a Body moves in the Directions AD, AF, from the Point A, by the Force of its Gravity, (FIG. V.) *the Magnitude of the Stroke made with the oblique Direction AD, ſhall be to that made with the Direction AF, perpendicular to the Horizon, as AF : AD.*

FOR (by the foregoing *Lemma*) the Strokes made with the oblique and perpendicular Directions, are the ſame as if the Body had moved uniformly all along, with a Degree of Velocity equal to that gotten by the accelerated Motions, at the Points where the Lines of Direction meet the Plane. But the Velocities gotten (by the Deſcents) at the Points F and D, where the Lines of Direction meet the Plane, are equal (by what is ſhewn in *Galilæus*'s 3d *Mechanick Dialogue*. Therefore the Strokes are the ſame, as if they were made in thoſe Directions, with equal Veolcities in uniform Motions. And therefore (by what has been already ſhewn) the Stroke by AD, is to that by AF, as AF : AD. Q:E:D.

C o r.

Therefore where the Body deſcending in the perpendicular Direction AF, acquires a Degree of Velocity, as much leſs than that which it acquires by the Deſcent thro' the whole Line AF, (or which is all one, by the Deſcent thro' AD) as AF is leſs than AD; I ſay, where it acquires ſuch a Degree of Velocity; there the direct Stroke made upon a Plane drawn thro' that Point, parallel to FD, ſhall be equal to the oblique Stroke made in the Direction AD upon the Plane FD; the Body in both Caſes moving freely by the Force of its Gravity, as was ſuppoſed in the *Theorem*. This is demonſtrated in all Reſpects like the 9th *Coroll.* of the foregoing Problem.

From hence we may eaſily ſolve the following Problem.

But it is $W : v :: \sqrt{AF} : \sqrt{AH}$ $\left\{\begin{array}{l}\text{From the } \textit{Law} \\ \textit{of Gravity.}\end{array}\right.$

And V is $= W$. $\left\{\begin{array}{l}\text{By } \textit{Galilæus's} \\ \textit{Demonſtration.}\end{array}\right.$

Therefore it is alſo $V : v :: \sqrt{AF} : \sqrt{AH}$
Therefore it muſt be $\sqrt{AF} : \sqrt{AH} :: AF : AC$
That is, it muſt be $AF : AH :: AF^q : AC^q$
or (if $AE = AC$) it muſt be $AF : AH :: AF^q : AE^q$
Therefore putting $AE = AC$, the Line AF muſt be divided ſo, that AF, AE and AH may be continual Proportionals. Q: E: I.

THEOR. II.

If a Body deſcending by the Force of its Gravity (in any Direction, whether perpendicular or oblique) ſtrikes upon parallel Planes; the Magnitudes of the Strokes (whether direct or oblique) ſhall be directly proportional to the Times of the Body's Deſcent in thoſe Directions.

CASE I.

LET the Body deſcend in the Direction AF (*FIG.* V.) perpendicular to the Horizontal Lines HC, FD. The Strokes made upon the Planes HC, FD, after the Deſcents thro' AH, FD, are as the Velocities with which they are made; that is, as the Velocities gotten by thoſe Deſcents. But the Velocities (from the Nature of uniformly accelerated Motion) are proportional to the Times. Therefore, *&c.* Q: E: D.

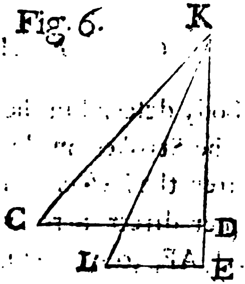
CASE II.

Let the Body deſcend in the oblique Direction AD. The Strokes made upon the Planes HC, FD, after the Deſcents thro' AC, AD, are to the former direct Strokes upon the ſame Planes as AH, AF, to AC, AD, by the foregoing Theorem. Therefore the oblique Strokes are directly proportional to the direct ones, ſince (from the ſimilar Triangles) AH : AF :: AC : AD. Therefore they are alſo as the Times of the perpendicular Deſcents, by the former *Caſe*. But (by *Theor.* III. of *Galilæus*'s 3d *Mechanick Dial.*) the Times of Deſcent thro' AC, AD, are to the Times of Deſcent thro' AH, AF; as the Lines AC, AD, to the Lines AH, AF. Therefore the oblique Strokes upon the Planes HC, FD, are alſo directly proportional to the Times of the Deſcent of the Body in the Directions AC, AD. Q: E: D.

COR.

The Strokes either oblique or direct, made upon the the Planes HC, FD, are as $\sqrt{AH}$, and $\sqrt{AF}$, or as $\sqrt{AC}$, and $\sqrt{AD}$; or, which is all one, they are as the Ordinates (of a Parabola) belonging to the *Abſciſſa* AH, AF, reſpectively.

Fig. 6.



THEOR. III.

A Body deſcending by the Force of its Gravity in Lines of Direction of equal Length, but unequally enclin'd to Two Planes; the Magnitudes of the oblique Strokes ſhall be to one another directly as the ſquare Roots of the Cubes of the perpendicular Altitudes of the Lines of Direction.

LET (*FIG.* VI.) KC=KL, and a Body deſcending in thoſe Directions, ſtrike upon the Horizontal Planes CD, LE. Draw KDE perpendicular to the Horizon, and conſequently ſo to each of thoſe Planes. Let D expreſs the Magnitude of the direct Stroke made upon the Plane LE, O the Magnitude of the oblique one, *d* the direct Stroke upon the Plane CD, and *o* the correſpondent oblique one.

Then $d : o :: KC : KD$
And $D : O :: KL : KE$ } By *Theor.* I.

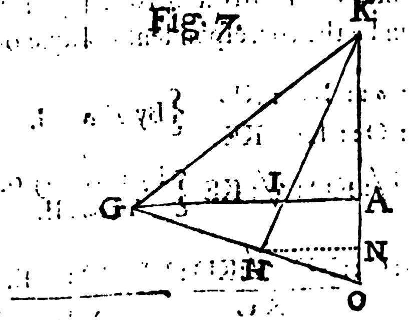
But $d : D :: \sqrt{:}\,KD : \sqrt{:}\,KE$ } By *Corollary* of *Theor.* II.

Therefore $o : O :: \frac{KD \times \sqrt{:}\,KD}{KC} : \frac{KE \times \sqrt{:}\,KE}{KL}$

But $KL = KC$, by the Suppoſition,

Therefore $o : O :: KD^{\frac{3}{2}} : KE^{\frac{3}{2}}$, that is directly as the ſquare Roots of the Cubes of the Altitudes of the Lines of Direction. Q. E. D.

Fig. 7.



THEOR. IV.

A Body deſcending by the Force of its Gravity in a Line of Direction, which is unequally enclin'd to Two Planes (meeting the ſame Point) the Magnitudes of the oblique Strokes given upon theſe Planes, ſhall be directly as the perpendicular Altitudes of the Line of Direction.

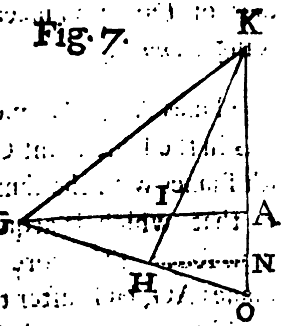
LET (*FIG.* VII.) the Body deſcend in the Direction KG, and ſo ſtrike upon the Planes GA, GH, making the Angle AGH. Let KA be perpendicular to the Horizontal Line GA; and KH be perpendicular to the enclin'd Line GH. The Strokes made by any accelerated Motions are the ſame as if they were made by equable Motions in the ſame Directions, and with Velocities equal to thoſe gotten

ten by the accelerated Motions, at the Points where the Lines of Direction meet the Planes (by the foregoing *Lemma.*)

But ſince the Planes AG, HG, meet the Line of Direction KG in the ſame Point G; the Body falls upon both Planes with the ſame Degree of Velocity, *viz.* that which is acquired by the Deſcent through KG. Therefore the Strokes made on the Planes AG, HG, after the Deſcent, are the ſame as if they were made in the ſame Direction, and upon the ſame Planes, with one and the ſame Degree of Velocity, in equable Motions. But if they were made with the ſame Velocities in equable Motions, the Stroke upon AG would be to that upon HG, as KA : KH (by *Cor.* IV. *Prob.* I.)

Therefore the Strokes are in the ſame Proportion now; that is, directly as the Altitudes of the Line of Direction above the Two Planes. Q: E: D.

Fig. 7.



THEOR. V.

A Body deſcending by the Force of its Gravity in Lines of Direction of unequal Length and Inclinations to a Plane, but equally high; the Magnitudes of the oblique Strokes ſhall be reciprocally as the Lengths of the Lines of Direction.

LET the Body (*FIG.* VII.) deſcend in the Directions KG, KI, and let the Angle AGH=IKG; then ſhall the Angle KGH=KIA.

Now becauſe the Velocity gotten by Deſcent thro' KI, is equal the Velocity gotten by Deſcent thro' KG, (by a foremention'd *Theor.* of *Galilæus's*) and becauſe the Angle KIA is = KGH; therefore the Stroke with the Direction KI upon GA, is equal the Stroke with the Direction KG upon GH: For the Lines of Direction

rection make the ſame Angles with the Planes, and the Body falls upon them with the ſame Degree of Velocity. But (by *Theor.* IV.) the Stroke with the Direction KG upon GA, is to the Stroke with the ſame Direction KG upon GH, as KA to KH; that is (becauſe of the ſimilar Triangles HKG, AKI) as KI to KG. Therefore alſo the Stroke with the Direction KG upon GA, is to that with the Direction KI upon GA, as KI to KG; that is reciprocally as the Lengths of the Lines of Direction. Q: E: D

THEOR. VI.

Univerſally; Any Two oblique Strokes given by a Body, deſcending by the Force of its Gravity, are in the Ratio *compounded of of the direct* Ratio *of the ſquare Roots of the Cubes of the Altitudes, and the reciprocal* Ratio *of the Lengths of the Lines of Direction.*

This is demonſtrated by the Third and Fifth Theorems.

N. B. In like manner we may demonſtrate Theorems for the Quantities of Strokes, when the Motions are perform'd according to any other Law of Acceleration; as when the Velocities are in the Duplicate, Triplicate, or any other Multiplicate or Sub-multiplicate *Ratio* of the Times: Concerning which ſee Dr. *Wallis*'s *Prop.* 5. *Cap.* 10. *De Motu Projectorum.* But ſuch Motions as theſe not being any where found in Nature, we'll paſs over that Speculation.

S C H O L. I.

The Proportions relating to the Buſineſs of Percuſſion, already prov'd in the *Corollaries* of *Prob.* I. may be aptly enough computed by a general *Calculus*, after this Manner. Let D. *d*. expreſs any direct or perpendicular Strokes. V. *v*. the Velocities with which they are made reſpectively. O. *o*. the correſpondent oblique Strokes, made with the ſame Velocities as the perpendicular ones. H. *h*. the Hypothenuſa's or longer Sides that expound the direct Strokes. P. *p*. the ſhorter Sides or Perpendiculars (from the Lines of the oblique Directions to the Planes upon which the Strokes are made) which expound the oblique Strokes. M. *m*. the Magnitudes or Bulks of the ſtriking Bodies, which we ſuppoſe to be Homogeneal. And let it be obſerv'd (which I mention here once for all) that all the Quantities expreſs'd by the great Letters do belong to one another, and ſo likewiſe thoſe expreſs'd by the little ones. Thus *d* expreſſes the direct Stroke made by the Body whoſe Magnitude is *m*, its Velocity *v*, the oblique Stroke made by the ſame *o*, the Line that expounds that direct Stroke h, that which expounds the oblique p. And ſo of the others.

It's obvious, that if $M=m$, then $D : d :: V : v$; and if $V=v$, then $D : d :: M : m$; ſo that univerſally $D : d :: M \times V : m \times v$; the Magnitudes of any Two direct Strokes are in the *Ratio* compounded of the direct *Ratio's* of the Magnitudes, and the Velocities of the moving Bodies. Therefore if $D=d$, then $M \times V = m \times v$, and $M : m :: v : V$; if the Magnitudes of the Bodies be reciprocally as the Velocities, the direct Strokes ſhall be equal. Again, $D : O :: H : P$, and $d : o :: h : p$, (by the general *Theor.* found at *Prob.* I.) but univerſally $D : d :: M \times V : m \times v$, therefore $M \times V : O :: H : P$, and $m \times v : o :: h : p$, and conſequently $O : o :: \frac{M \times V \times P}{H} : \frac{m \times v \times p}{h}$ univerſally. That is, the Magnitudes of any oblique Strokes are in the *Ratio* compounded of the direct *Ratio* of the perpendicular Strokes, the direct *Ratio* of the Aaltitudes, and the reciprocal *Ratio* of the Lengths of the Lines of Direction. Therefore if $M \times V = m \times v$, that is, if $D=d$, then $O : o :: \frac{P}{H} : \frac{p}{h} :: P \times h : p \times H$; that is, the oblique Strokes are in the *Ratio* compounded of the direct *Ratio* of the Altitudes, and the Reciprocal of the Lengths of the Lines of Direction; as was demonſtrated at *Cor.* VI.

If $P = p$, then $O : o :: \frac{1}{H} : \frac{1}{h} :: h : H$, which was ſhewn at *Cor.* V. If $H = h$, then $O : o :: P : p$, which was prov'd at *Cor.* III. If $\frac{P}{H} = \frac{p}{h}$, that is if $P : p :: H : h$, that is, if the Rectangular Triangles be ſimilar (and ſo the Angles of the Lines of Direction upon the Planes be equal) then $O : o :: (M \times V : m \times v ::) \; D : d$, as was demonſtrated at *Cor.* I. and exemplified in ſome particular Inſtances at *Cor.* VII. Laſtly, To find an oblique Stroke equal to a direct one made with a leſs Degree of Velocity, that is, the oblique Stroke O equal to the direct Stroke *d*. 'Tis clear, that $\frac{O}{d} = \frac{O}{D} \times \frac{D}{d}$; but $\frac{O}{D} = \frac{P}{H}$, and $\frac{D}{d} = \frac{M \times V}{m \times v}$, therefore $\frac{O}{d} = \frac{P \times M \times V}{H \; m \times v}$; conſequently $O : d :: P \times M \times V : H \times m \times v$ univerſally. Therefore if it be the ſame Body, that is, if $M = m$, then $O : d :: P \times V : H \times v$; conſequently, if $O = d$, then $P \times V = H \times v$, or $P : H :: v : V$. That is an oblique Stroke is = to a direct one, made with a Velocity as much leſs than that with which the Body moves in the oblique Direction; as the Perpendicular P is leſs than the Hypothenuſe H, which is *Cor.* IX.

To carry this on to Strokes made by Motions uniformly acclerated : Let us reaſſume the univerſal Analogy before laid down, *viz.* $D : d :: M \times V : m \times v$. Where it needs only be noted, that the Quantities V. v. which before expreſs'd the uniform Velocities in equable Motions, do now expreſs the Velocities gotten by the Deſcents (in accelerated Motions) at the Points where the Lines of Direction meet the Plane. By *Theor.* I. theſe Analogies hold good in the Caſe of accelerated Motions too ; *viz.* $D : O :: H : P$, and $d : o :: h : p$. And therefore (as was ſhewn in the Caſe of equable Motions in the foregoing *Calculus*) $M \times V : O :: H : P$, and $m \times v : o :: h : p$. But (from the known Law of Gravity) 'tis $V : v :: P^{\frac{1}{2}} : p^{\frac{1}{2}}$ (the Velocities are in the Subduplicate *Ratio* of the Lines of Deſcents) therefore $M \times P^{\frac{1}{2}} : O :: H : P$, and $m \times p^{\frac{1}{2}} : o :: h : p$. Therefore $O : o :: \frac{M \times P^{\frac{1}{2}}}{H} : \frac{m \times p^{\frac{1}{2}}}{h}$ univerſally. That is, the Magnitudes of any Two oblique Strokes are in the *Ratio* compounded, of the direct *Ratio* of the Bodies, the direct *Ratio* of the $\frac{1}{2}$ Powers of the Altitudes, and the reciprocal *Ratio* of the Lengths of the Lines of Direction. Therefore if $M = m$, then $O : o :: \frac{P^{\frac{1}{2}}}{H} : \frac{p^{\frac{1}{2}}}{h}$; which was the Sixth *Theorem* forgoing. If $P = p$; then $O : o ::$

$\frac{1}{H} : \frac{1}{h}$, as was demonſtrated at *Theor.* V. If $H = h$, then $O : o :: P^{\frac{1}{2}} : p^{\frac{1}{2}}$, as was ſhewn at *Theor.* III. If $\frac{P}{H} = \frac{p}{h}$, that is, if $P : p :: H : h$ (that is if the Rectangular Triangles be ſimilar, and conſequently the Angles of the Lines of Direction upon the Plane be equal) then it will be $O : o :: P^{\frac{1}{2}} : p^{\frac{1}{2}}$, or $O : o :: H^{\frac{1}{2}} : h^{\frac{1}{2}}$, which was the *Coroll.* of *Theor.* II. Alſo becauſe $V : v :: P^{\frac{1}{2}} : p^{\frac{1}{2}}$, therefore $O : o :: V : v$ but (putting T, t, the Times of Deſcent) 'tis $V : v :: T : t$ (from the Nature of uniformly acclerated Motion) therefore $O : o :: T : t$, which was Caſe II. of *Theor.* II. and the firſt Caſe will be as eaſily deduced in like manner. Laſtly; To find an oblique Stroke equal to a direct one made with a leſs Degree of Velocity, I proceed altogether as in the *Calculus* for the like Caſe, in the Hypotheſis of uniform Motions, and ſo come in general Terms to this Concluſion (there inveſtigated) that $O = d$, when $P \times V = H \times v$; that is, when $P : H :: v : V$. And this is the *Corollary* to *Theor*: I. about Motions uniformly acclerated. And why I may by the very ſame Steps advance towards the Concluſion, in the Caſe of acclerated Motions, that I did in the Caſe of equable Motions, I dare preſume is evident to any one, that conſiders but the foregoing *Lemma*, and the *Theorem* that immediately ſucceeds it.

But now to work more particularly and directly to determine whereabouts a heavy Body in its perpendicular Deſcent ſhall acquire ſuch a Degree of Velocity, as to make a direct Stroke equal to an oblique one; that is, where O ſhall $=d$. It's plain, that $\frac{O}{d}=\frac{O}{D}\times\frac{D}{d}$; but (by *Theor.* I.) $\frac{O}{D}=\frac{P}{H}$, and $\frac{D}{d}=\frac{V}{v}$ (by what has been ſhewn already) and $\frac{V}{v}=\frac{P^{\frac{1}{2}}}{p^{\frac{1}{2}}}$ (by the Law of Gravity) Therefore $\frac{O}{d}=\frac{P}{H}\times\frac{P^{\frac{1}{2}}}{p^{\frac{1}{2}}}=\frac{P^{\frac{3}{2}}}{H\times p^{\frac{1}{2}}}$; conſequently $O:d::P^{\frac{3}{2}}:H\times p^{\frac{1}{2}}$. Therefore when $O=d$. then $P^{\frac{3}{2}}=H\times p^{\frac{1}{2}}$, and ſo $P:H::p^{\frac{1}{2}}:P^{\frac{1}{2}}$. Therefore if (*FIG.* V.) we put $AF=P$, and $AD=H$, then p ($=AH$ the Length in the Perpendicular that is ſought) muſt be ſuch, that $AF:AD::\sqrt{AH}:\sqrt{AF}$. But $AF:AD::AC:AF$, therefore AH or p muſt be ſuch, that $AC:AF::\sqrt{AH}.\sqrt{AF}$. That is (putting $AE=AC$) we muſt have $AEq:AFq::AH:AF$, or $AEq=AF\times AH$, and ſo the Lines AF. AE. AH muſt be continued Proportionals; which was the Conſtruction before propos'd. And the Amount of it is no more than to let fall the Perpendicular CH from the Point C, which determines the Point H that was ſought; ſince 'tis certain (from the ſimilar Triangles) that $AF:AC::AC:$ (or AE) AH. Q: E: I.

Ha-

SCHOL. II.

Having at *Theor*. II. made ſome Compariſon between the Times of a heavy Body's Deſcent, and the Magnitudes of the Strokes made upon a Plane after thoſe Deſcents; it may not be amiſs to purſue that Matter a little further. All the Symbols ſtanding as in the foregoing *Scholium*; let T, t, expreſs Timesof Deſcent, in the oblique Lines H. h.

1. 'Tis certain, that when $P=p$, then $O:o::h:H$ } By *Theor*. 5.
But alſo when $P=p$, then $T:t::H:h$ } By *Theor*. 3. *Galil.* 3d *Dial.*
Therefore in this Caſe the Quantities of the oblique Strokes are reciprocally as the Times of Deſcent.

2. If $H=h$, then $O:o::P^{\frac{3}{2}}:p^{\frac{1}{2}}$ } By *Theor*. 3.
But alſo when $H=h$, then $T:t::p^{\frac{1}{2}}:P^{\frac{1}{2}}$ } By *Theor*. 4. *Galil.*
Therefore in this Caſe the Quantities of the obliqne Strokes are reciprocally as the Cubes of the Times of Deſcent.

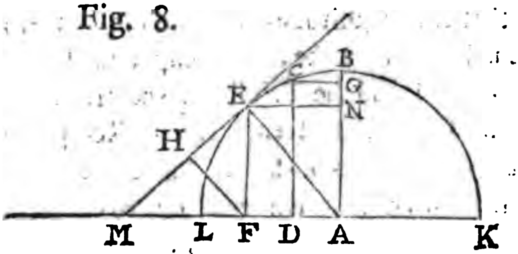
3. When neither $H=h$, nor $P=p$, then univerſally $O:o::h\times P^{\frac{3}{2}}:H\times p^{\frac{3}{2}}$ } By *Theor*. 6.
But 'tis alſo qniverſally, $T:t::H\times p^{\frac{1}{2}}:h\times P^{\frac{1}{2}}$ } By *Theor*. 5. *Galil.*

So that the Magnitudes of the oblique Strokes are univerſally in the *Ratio* compounded, of the Reciprocal, and the Triplicate Reciprocal *Ratio* of the Times of Deſcent; that is, Reciprocally as the Biquadrates of them. And this Rule may be laid down as an univerſal one to expreſs the Proportion in Lines, *viz.* $\frac{O}{o} : \frac{T}{t} :: \frac{h \times P^{\frac{3}{2}}}{H \times p^{\frac{3}{2}}} : \frac{H \times p^{\frac{1}{2}}}{b \times P^{\frac{1}{2}}} :: h^2 \times P^2 : H^2 \times p^2$; or $O \times t : o \times T :: h^2 \times P^2 : H^2 \times p^2$.

SCHOL. III.

The Quantities of Strokes made upon Curves, may be computed as well as thoſe upon right Lines. For the Stroke upon any Point of a Curve is the ſame with that upon a Tangent at the ſame Point: Since the Declivity of a Curve and its Tangent are the ſame at the Point of Contact. *Prop.* 15. *Cap.* 2. of Dr. *Wallis*'s *Mechanicks.* Upon this Head therefore I think it not improper to ſubjoin a *Theorem* or two.

Fig. 8.



THEOR. VII.

*In a Circle, the oblique Stroke made upon the Curve at any Point as E (*FIG. VIII.*) with the Direction of the Ordinate to that Point FE; is to a direct Stroke made at the ſame Point, and with the ſame Velocity (which I every where ſuppoſe) as the Ordinate FE to the* Radius *EA.*

LEtting fall FH perpendicular from F to the Tangent ME; 'tis plain by what has been ſhewn, that the oblique Stroke upon the Plain ME, with the Direction FE, is to the direct one as FH : FE. But the Stroke upon the Tangent, and the Curve is all one at the Point of Contact E; the oblique and direct Strokes therefore upon the Curve are as FH and FE. But the Triangles FHE, AFE are ſimilar; and FH : FE :: FE : EA. Therefore, *&c.* Q: E: D.

COR.

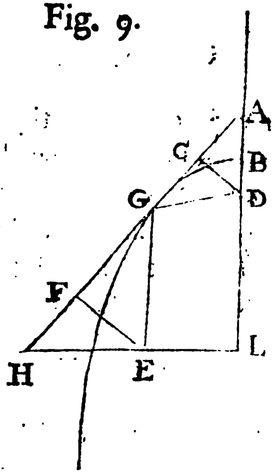
COR. I.

The oblique Stroke upon the Curve at E, with the Direction FE, is to the direct Stroke at the ſame Point; as the oblique Stroke upon the Plain at A, with the Direction EA, is to the direct Stroke at the ſame Point A. For both Proportions are that of FE : AE.

COR. II.

The oblique Strokes made with the ſame Velocity upon the ſeveral Points of the Curve, and with the Directions of the Ordinates; are one to another directly as thoſe Ordinates, or as the Sines of the coreſpondent Arches. Thus the oblique Stroke at E, is to that at C, as the Sine of the Arch LE, to the Sine of the Arch LC.

Fig. 9.



THEOR. VI.

The oblique Strokes made upon the Curve of a Parabola, with the Directions of the Diameters of that Parabola; are ever in the Subduplicate Ratio *of the direct Strokes.*

LEt GE be any Diameter of the Parabola BGH (*FIG.* IX.) whoſe Axe is BL, Vertex B, Focus D. Conceive a Stroke made in the Direction EG, upon the Curve at G, or which is all one, upon the Tangent GA at that Point. Draw DC perpendicular from the Focus to the Tangent; likewiſe EF perpendicular to the ſame from the Point E.

By what has been hitherto ſhewn, the Magnitude of the oblique Stroke at G is to the Magnitude of the direct one, as FE : GE. But from the Nature of the Parabola, the Angle FGE is always = CGD, and thoſe at F and D are right ones by the Conſtruction ; therefore the Triangles FGE, and CGD are always ſimilar. Therefore ſince FE : GE : : CD : DG, the Magnitude of the oblique Stroke, is to that of the direct always as CD : DG. But from the Nature of the Parabola DG is ever as CD^q, that is the Diſtance of any Point in the Curve from the Focus, is ever as the Square of the Perpendicular from the Focus to the Tangent at that ſame Point (which Property our *Celebrated Author* demonſtrates at *Lem.* 14. *Sect.* 3. *Lib.* 1. and which we ſhall afterwards give the Inveſtigation of.) Therefore the oblique Strokes are ever in the Subduplicate *Ratio* of the direct ones. Q : E : D.

COR. I.

'Tis manifeſt that the ſame Proportion holds with reſpect to the oblique Strokes made with the Directions of Lines, drawn out of the Focus to any Points in the Curve.

COR. II.

If the direct Strokes be made with the ſame Velocity and ſo be equal, the oblique ones ſhall be equal all over the Parabola. For they are ſtill in the ½ *Ratio* of the direct ones; and to be ever in any certain *Ratio* of Equals, is to be always equal.

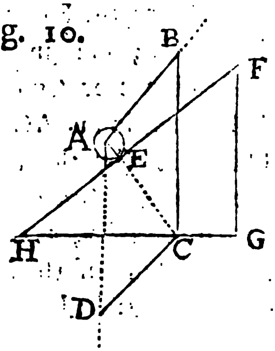
THEOR. IX.

If the Curve in FIG. IX. *were either of the other* Conick Sections, *one of whoſe* Foci *were D, as before, and the other* Focus *were at the Point E (falling either within or without the Section) the Proportion of the direct and oblique Strokes made upon the Curve wou'd be as before,* viz. *as the Diſtance from either* Focus *to any Point in the Curve is to a perpendicular from the ſame* Focus, *let fall upon the Tangent at that Point.*

THIS needs no Demonſtration. But then, as in the Ellipſes, the Perpendicular from the *Focus* to a Tangent at any Point is ſtill in leſs, and in the Hyperbola ſtill in more, than the ½ *Ratio* of the Diſtance of that Point from the *Focus* (of which we may have Occaſion to ſpeak more hereafter) ſo in the former Curve will the oblique Strokes (made with the Directions of Lines drawn from the *Focus*) be ſtill in leſs, and in the Hyperbola ſtill in more, than the $\frac{1}{2}$ *Ratio* of the direct ones.

But I ſhall now proceed to the Explication of the *Mechanick* Powers.

Fig. 10.



Prob. II.

A Body being ſuſtain'd by a Power (acting with any given Direction) upon an enclin'd Plane; 'tis required to find the Proportion of the Power to the Weight of the Body.

LET the Weight A (*FIG.* X.) be held upon the enclin'd Plane HF, by a Power acting with the Direction AB. Let AE be perpendicular from A (the Center of the Body) to the enclin'd Plane, and produce it to C in the Horizontal Line HG. Draw CB (perpendicular to HG, or parallel to FG) to cut the

Line of Direction in B: And Laſtly, (to compleat the Parallelogram) draw the Lines CD and AD, parallel to AB and BC reſpectively. The Body now is to be conceiv'd as ſuſtain'd by Three Powers (one of which is equivalent to the other Two) one, the Power, whoſe Direction is AB; the other, the Force of Gravity, whoſe Direction is AD; and (in the Room of) the third, the Reſiſtance or Contranitence of the Plane, whoſe Direction is AC perpendicular to it. Therefore (by the third general *Theor.* after the *Laws of Motion*) the Power at B is to the Force of Gravity, as the Sine of the Angle CAD, to the Sine of the Angle ACD; that is, as CD : DA. Q: E: I.

COR. I.

If the Line of Direction AB were parallel to the enclin'd Plane HF, then the Angle CAB would be a Right one, for AC is perpendicular to HF. Conſequently the Rectangular Triangle CAB, will be ſimilar to the Rectangular Triangle FGH. From thence AB : BC :: (that is, CD : DA ::) FG : FH; ſo that the Power is to the Force of the Weight, as the Sine of the Angle of Elevation (FHG) to the *Radius*.

COR. II.

If the Line of Direction AB were parallel to the Baſe HG, then the Angle DAB would be a Right one; and conſequently CD : DA :: FG : HG; and ſo in this Caſe the Power is to the Force of the Weight, as the Sine of the Angle FHG, to the Sine of the Angle HFG.

COR. III.

That Power which ſuſtains a Body upon an enclin'd Plane, having its Line of Direction parallel to the Plane; is leſs than the Power which ſuſtains the ſame Body upon the ſame Plane, with a Direction parallel to the Baſe. For in the former Caſe the Proportion is as FG : FH, and in the latter as FG : HG. And 'tis evident that 'tis as much leſs, as HG is leſs than FH.

COR. IV.

The Force by which the Body endeavours to deſcend upon the enclin'd Plane, is to the Force by which it endeavours to deſcend in a Perpendicular, as CD : DA univerſally (which may be eaſily determin'd to the particular Caſes of *Cor.* I. and II.) For the Power is to the abſolute Force of the Weight (that by which it endeavours to deſcend in a perpendicular Direction)

rection) as CD : DA, by what has been ſhewn. But the Power ſuſtains, (and therefore is equal to) the Force by which it endeavours to deſcend upon the enclin'd Plane. Therefore theſe Forces of the Body are alſo in the ſame *Ratio*.

COR. V.

From the ſame Principles we may compute the Proportions of the Power to the Gravity of the Weight; when the Inclination of the Plane (upon which the Weight is ſuſtain'd) is continually changing; that is, when we imagine the Body to be held ſucceſſively upon the ſeveral Points of any Curve Surface; in which Caſe the Tangents to thoſe Points, are the changing enclin'd Planes that we are to conſider.

Thus we may conceive the Line HF (*FIG*. X.) to be the Tangent of ſome Curve (whoſe Vertex is in ſome Point between F and G) FG the Subtangent, and HG an Ordinate from the Point of Contact H. 'Tis evident, that to ſuſtain the Weight A continually upon the Curve, there muſt be a different Power for every different Point where it reſts. And the Proportion of the Power to the Gravity of the Weight; for every different Point will be as the Subtangent to the Tangent, in the Caſe of *Cor*. I. or as the Subtangent to the Ordinate, in the Caſe of *Cor* II. Therefore applying the Subtangents and the Tangents, or the Subtangents geuts, and the Ordinates, to ſome common Axe; the Curvilineal Figures thence ariſing, will give the Summatory *Ratio* of the Powers to the Force of Gravity, for all the Points of Application thro' the Curve, the *Pondus* reſts upon. *Ex. gr.*

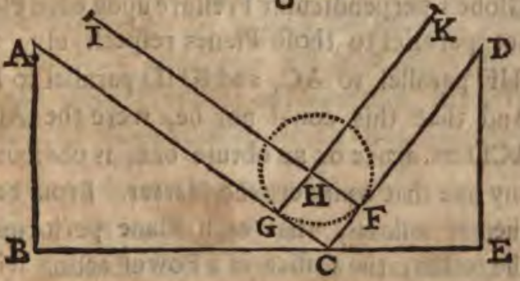
If FH were the Tangent of a Parabola, whoſe Parameter $= r$, and Abſciſſe $= x$. The Subtangent than will be $= 2x$, and the Tangent $= \sqrt{rx + 4xx}$, or taking the Halfs of both, we have x, and $\sqrt{\frac{r}{4}x + xx}$, for the Ordinates of Two Curves. Applying theſe continually therefore to the Abſciſſe (x) of the Parabola, we ſhall have an Iſoſceles Triangle, and an equilateral Hyperbola; for 'tis plain that $\sqrt{\frac{r}{4}x + xx}$ is the Expreſſion of the Ordinate of an equilateral Hyperbola, whoſe Tranſverſe, Conjugate or Parameter are either of them equal to $\frac{r}{4}$. If the Line of Direction were parallel to the Baſe HG, which is the Ordinate; then the Curve being ſuppoſed a Parabola, as before, the Two emerging *Areas*, would be an Iſoſceles Triangle, and that very ſame Parabola, both having the ſame Abſciſſe $(x.)$ And in either Suppoſition the *Areas* thus made expreſs the *Ratio Totalis* of the ſuſtaining Powers to the Force of Gravity.

mogeneal Bodies are proportional to their Magnitudes or Bulks) therefore $M\times A=m\times L$, and ſo $M : m :: L : A$. That is, a Body as much leſs than the Body propoſed, as the Height of the enclin'd Plane is leſs than the Length of it, will be that requir'd. Q: E: I.

If the *Momentum* of a Body held upon an enclin'd Plane were not to be equal, but in any *Ratio* aſſign'd, to the abſolute *Momentum* of another Body, *Ex. gr.* in the *Ratio* of $p : q$. Then it wou'd be $\frac{G\times A}{L} : g :: p : q$, and $g\times p=\frac{G\times A\times q}{L}$, that is, $m\times p=\frac{M\times A\times q}{L}$, from whence $M : m :: p : \frac{A\times q}{L} :: L\times p : A\times q$. That is, the Bodies muſt be in the *Ratio* compounded of the direct *Ratio* of L to A, and the Direct of p to q. *Ex. gr.* Suppoſe $p : q :: G : \frac{G\times A}{L}$ (that is, as the abſolute *Momentum* of the Body M, to its *Momentum* upon the enclin'd Plane. Then $\frac{G\times A}{L} : g :: G : \frac{G\times A}{L}$, that is, $G : \frac{G\times A}{L} :: \frac{G\times A}{L} : g$, in which Caſe the *Momentum* of the Body M upon the enclin'd Plane, comes to be a mean Propotional between its own abſolute *Momentum* and the abſolute *Momentum* of the Body *m*. Therefore $G\times g=\frac{G^2\times A^2}{L^2}$, that is, $g\times L^2=G\times A^2$, or by equivalent Subſtitution,

$m \times L^2 = M \times A^2$; from whence $M : m :: L^2 : A^2$ That is, if the Body M be to the Body *m*, as the Square of the Length of the Plane, to the Square of the Height; then the *Momentum* of the Body M upon the enclin'd Plane ſhall be a mean Proportional, between its own abſolute *Momentum*, and the abſolute *Momentum* of the Body *m*; or *Vice Verſâ*.

Fig. 11.



PROB. II.

A Globe reſting upon Two enclin'd Planes, which cut one another at Right Angles; 'tis requir'd to find what Proportion of the Preſſure, or Burden each Plane ſuſtains.

LET the Globe H reſt upon the Two enclin'd Planes (*FIG.* XI.) AC and CD; and let the Angle ACD be a Right one. Uſing the Symbols as in the foregoing *Schol.* let G expreſs the abſolute Gravity, or total *Momentum* of the Globe, which is ſhared betwixt the Two Planes (each of which bears its Part) but in what Proportion, we are now to determine. Let AC=L, AB=A : DC=*l*, DE=*a*, according to the Intent of the *Schol.* The Angle ACD made by the Two Planes, being a Right one by the Hypotheſis, 'tis evident, that the Quadrilateral Figure GHFC is a perfect Square ; or, which is all one, that the Lines IHF, KGH, which are the Lines of the Direction of the Globe's perpendicular Preſſure upon each Plane, are parallel to thoſe Planes reſpectively ; *viz.* IHF parallel to AC, and KHD parallel to DC. And that this cou'd not be, were the Angle ACD an acute or an obtuſe one, is obvious to any one that conſiders the Matter. From hence then it follows, that each Plane performs to the other, the Office of a Power acting with a Direction parallel to the Plane. Thus the Plane AC ſuſtains all that Force of the Globe, by which it endeavours to deſcend upon the enclin'd Plane DC, and which Force, if the Plane AC were taken away, or Powers (acting with the Direction KG parallel to the Plane DC) wou'd ſuſtain. So likewiſe the Plane DC, ſuſtains the *Momentum* of the Globe upon the Plane AC, which *Momentum* wou'd be ballanc'd by a Power acting with the Direction IF parallel to AC, if the Plane DC were remov'd. But beſides theſe Preſſures, each Plane is burden'd with one of another kind, and that is the Difference between the total abſolute *Momentum* of the Globe and that relative or partial *Momentum* with which the Globe endeavours to deſcend upon that Plane. For if either of the Planes were taken away, and a Power was ſubſtituted in the room of it; then tho' that Power ſuſtain'd the Burden which preſs'd upon the Plane before, yet the Body ſtill reſting upon the other Plane, that Plane is preſs'd by it; and the Quantities of that *Momentum* by which the Plane is preſs'd is evidently what was juſt now aſſerted. To ſtate theſe Proportions therefore, 'tis certain that the *Momentum* by which the Globe endeavours to deſcend upon the Plane DC, is $=\frac{G\times a}{l}$; this the Plane AC ſuſtains. And the *Momentum* by which the Globe endeavours to deſcend upon the Plane AC, is, $=\frac{G\times A}{L}$; this the Plane DC ſuſtains. But the Plane DC ſuſtains alſo the *Momentum* $G-\frac{G\times a}{l}$; and the Plane AC ſuſtains alſo the *Momentum* $G-\frac{G\times A}{L}$. Therefore the whole *Momentum*, with which the Plane DC is burden'd, is $=\frac{G\times A}{L}+\frac{G\times l-G\times a}{l}$; and the whole *Momentum* with which the Plane AC is burden'd, is $=\frac{G\times a}{L}+\frac{G\times L-G\times A}{L}$. And the Sum of theſe Two Aggregates of *Momenta* (with which both the Planes are burden'd) caſting off Contradictories, will be $=\frac{G\times l\times L+G\times l\times L}{L\times l}=2G=$ to twice the abſolute *Momentum* of the Globe; which is abſurd and impoſſible. For the abſolute *Momentnm* of the Globe is parted between the Two Planes; and what they ſuſtain cannot poſſibly exceed that. We are therefore (as the excellent Mr. *Leibnitz* ſhews *Act. Lipſ.* for *Novemb.* 1685.) to take the Halfs of theſe Sums of *Momenta*, or (which amounts to the ſame) an Arithmetical Mean betwixt them. And then they will be $\frac{G\times A}{2L}+\frac{G\times l-G\times a}{2l}$, and $\frac{G\times a}{2l}+\frac{G\times L-G\times A}{2L}$; the Sum of which is $=G$, the abſolute *Momentum* of the Globe. The *Momentum* of the Globe then, with reſpect to each Plane, is determin'd. Q: E: F.

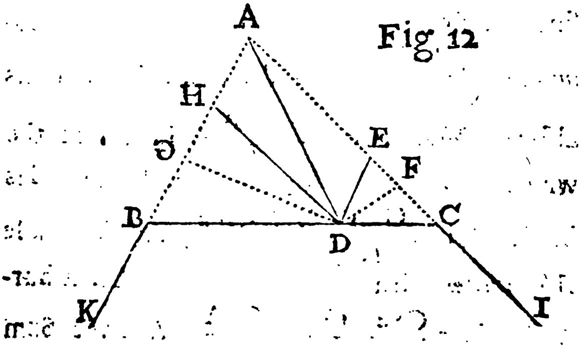


Fig. 12

PROB.

PROB. III.

Two Powers ſuſtaining each other upon a Vectis, 'tis requir'd to find the Proportion of one to the other.

LET BC be a Vectis, (*FIG.* XII.) whoſe Fulciment is D, and let the Powers K and I, acting with the Directions KB, IC, ſuſtain each others Forces,or be in *Equilibrio* about the Fulciment D. Produce the Lines of Direction till they meet in A, and draw AD: Likewiſe draw DE, DH parallel to the Lines of Direction, and DF, DG perpendicular to them. The Fulciment, or fix'd Point D, is here in the ſtead of a Third Power, and its Line of Direction is DA, drawn from D to A, the Concourſe of the Lines of Direction of the other Two Powers I and K. The Vectis therefore being held ſteady by theſe Three Powers; Two of them (which draw it with the Directions from A to B, and from A to C) deſtroy the Effect of the Third, which is to be imagined to draw it with a Direction from D to A. For the Reſiſtance of the Fulciment D (as the Vectis is preſs'd down upon it by the other Two Powers) is as much as another Power drawing it upwards, with a Direction from D to A, the Concourſe of the other Two Directions. Therefore (by *Cor.* II. of *Theor.* III.)

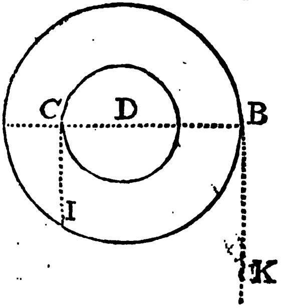
Power I : Power K : : Sine Angle HAD : Sine Angle EAD (or HDA) that is, as HD : ED. But now the Triangles HDG, EDF are ſimilar; for the Angles at G and F are Right ones by the Conſtruction, and becauſe of the Parallelogram, AED=AHD, and conſequently FED=GHD; therefore the Triangles are ſimilar. Therefore HD : ED : : GD : FD, and conſequently, Power I : Power K : : GD : FD; that is, the Powers are reciprocally as the Diſtances of their Lines of Direction from the Fulciment. Q : E : I.

C O R. I.

If the Lines of Direction BK and IC were parallel to each other, and perpendicular to the Horizon, then GD and DF would be coincident with DB and DC. Therefore then the Powers wou'd be reciprocally as the Diſtances (of the Points of Application) from the Fulciment. When the Lines of Direction of the Powers are parallel in the Vectis, then if inſtead of Two Powers, there were Two Weights appended to the Points B, C, the Machine wou'd become a common *Libra*, the Law of which would be as in the foregoing *Corollary*, viz. The *Pondera* (being in *Equilibrio*) are reciprocally as their Diſtances from the Center of the *Libra*. Q: E : I.

From

Fig. 13.



From the Explication of the Vectis we may without any further Trouble, deduce the Property of the Wheel, or the Axis in *Peritrochio*. For if DC be the *Radius* of the Axis to which a *Pondus* is applied in C; and DB the *Radius* of the Wheel, or outer Circle it ſelf, to which a Power is applied in B. The Line CB then may be conſider'd as a Vectis, of which the Fulciment is D; and the Power ballancing the Weight, that is, the Machine being held perfectly ſteady and immovable, the Proportion of the Power to the Weight will be always as the Diſtances of the Lines of Direction from the Fulciment D, reciprocally; that is, as DB, CD reciprocally, or, which is all one, thoſe Diſtances being the *Radij*, they will be as the Peripheries of the Wheel and the Ax reciprocally. Q: E: I.

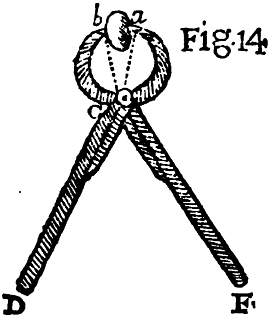
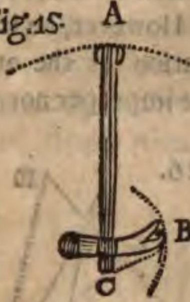


Fig. 14

From theſe Principles are alſo to be deduced the Powers of thoſe Vulgar Machines, Sciſſars, Pinchers, Sheers, Forcipes, the Crane, the Gimblet, the Hammer (as uſed and applied to the drawing out of a Nail) with various others. The Four firſt are manifeſtly a Pair of Vectes, placed croſs-wiſe, and interſecting each other in their common *Fulcrum*. Thus if *ab* CDE repreſented a Machine of that kind, where *ba* is the Obex or Body to be cut or compreſs'd, then DC and EC being imagin'd to be produced directly to *a* and *b*, we have here Two Vectes in a croſs Poſition, *viz.* DC*a*, and EC*b*, the common *Fulcrum* of which is C (the Center of the Rivet) and the moving Powers being applied at D and E, the other Extremities *a*, *b*, are applied to the Obſtacle. From whence the Power of this ſort of Machine is eaſily eſtimated. The Body and the moving Powers will ſuſtain one another, when the Power is, to the Difficulty or Re-

Reſiſtance of the Obſtacle, as the Diſtance of each reciprocally from C. As for the latter Machines, they are more immediately reducible to the Axis in *Peritrochio*, tho' ultimately reſolv'd into the Vectis: The Power of the Hammer is, perhaps, equally explicable either Way.

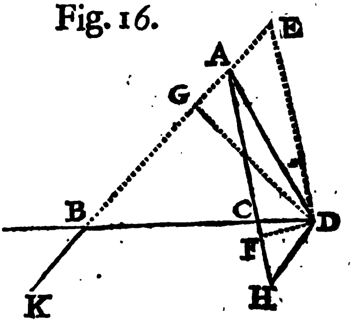
Fig. 15.



Thus the Power (*FIG.* XV.) being applied in A, the Extremity of the Handle, and the Obſtacle to be remov'd or drawn out in B, then the End C is the *Fulcrum* or Center of Motion. And the Power moving in a Circumference, whoſe *Radius* is CA and the Obſtacle (when mov'd) in another, whoſe *Radius* is CB. Hence if the Forces (of the Power, and of the Reſiſtance of the Obſtacle) are reciprocally, as thoſe Peripheries, or their reſpective *Radij* (which are the Diſtances from the Fulciment) they will ſuſtain one another, and be in *Equilibrio*.

The Vectis of the firſt ſort has been hitherto explain'd; and that partly in it ſelf, and partly the Machines, that are nearly related to it. For ſuch are all thoſe that have been mention'd under this Head, unleſs any one thinks it more proper to reduce the Hammer (applied to the Uſes juſt now mention'd) to the Principle of the recurv'd Vectis. However, be it which way it will, the Explication of the other kinds of Vectes will not be improper nor unuſeful.

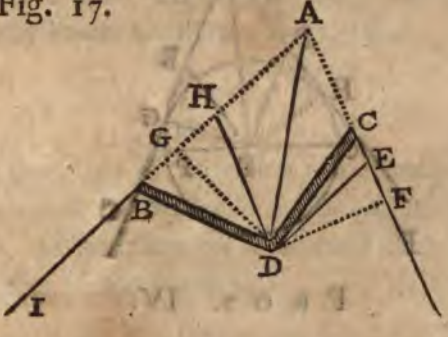
Fig. 16.



Let (*FIG.* XVI.) D be the Fulciment of the Vectis BCD, and the Powers at K and H, whoſe Lines of Direction are CB and HC, be conceiv'd to ſuſtain one another upon it. Produce the Line KB till it cuts HC in the Point A, and draw DE parallel to CA, till it meets the Line KBA produc'd in the Point E. Compleat the Parallelogram AEDH, and draw the Diagonal AD. Laſtly, From the Point D, let fall DG, DF perpendicular to the Lines of Directions

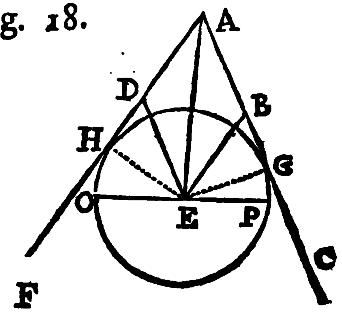
KBA and ACH. It will be prov'd all together in the ſame Manner as in the Caſe of the former Vectis; that the Power at H, is to the Power at K, as ED : DH; that is (becauſe the Rectangular Triangles DGE, DFH, are ſimilar) as DG : DF. Therefore when the Lines of Direction AK, AC are parallel, and to the Perpendiculars DG, DF coincide with DB, DC; then the Power will be reciprocally as thoſe Lines; that is, reciprocally as the Diſtances of the Points of Application from the Fulciment D.

Fig. 17.



If inſtead of either of the former, we had the incurv'd Vectis BDC (*FIG.* XVII.) propoſed, in which the Fulciment is at the Point D, and the Powers applied at B and C (acting with the Directions BI, CF) ſuſtain one another; the Property of this Machine will be found as eaſily from the ſame general Principle. For producing the Lines of Direction till they interſect in A, and then compleating the Parallelogram HADE, whoſe Diagonal is AD: And, Laſtly, Letting fall the Perpendiculars DG, DF, upon the Lines of Direction, 'twill be prov'd, that the Powers are as thoſe Perpendiculars reciprocally. Conſequently when the Lines of Direction are parallel, the Powers will be reciprocally, as the Diſtances of the Points of Application from the Fulciment D, taken in a Line parallel to the Horizon.

Fig. 18.



PROB. IV.

Two Powers acting with given Directions, ſuſtaining one another upon a Trochlea; it is requir'd to find the Proportion of thoſe Powers.

Suppoſe the Trochlea (*FIG.* XVIII.) firmly fix'd by its Center E to ſome Body that ſupports it, without any Motion. And let the Powers at F and C, whoſe Directions are FD, CB, ſuſtain one another upon it. Produce the Lines of Direction till they meet in A, and compleat the Parallelogram ADEB, and let fall the Perpendiculars EG, EH to the Lines of Direction, which being Tangents at H and G, theſe Perpendiculars will be *Radij*, from the Nature of the Circle. Therefore now (by *Cor.* II. of *Theor.* III.) Power at C : to Power at F : : as Sine Angle AEB : to Sine Angle BAE ; that is, as AB : EB, that is, as ED : EB. But the Triangles EGB, EHD are ſimilar (for the Angles at G and H are Right ones, and EBG = EDH, becauſe of the Parallelogram) therefore ED : EB : : HE : GE ; and conſequently Power C=Power F. Q : E : I.

COR.

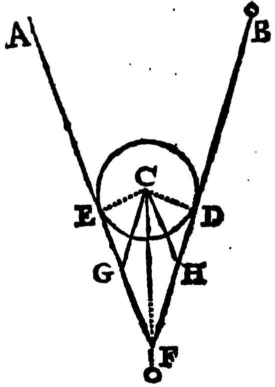
If the Lines of Direction were parallel, the Law would be the ſame, as alſo in this latter Caſe, if a Weight were ſubſtituted in the room of either of the Powers.

SCHOL.

Becauſe the Tangent AH, AG, are equal from the Nature of the Circle, therefore Angle HAE= GAE ; and ſo 'tis preſently to be concluded that Power C is = Power F. But to uſe the perpendiculars from the Center to the Lines of Direction

rection is not only equally clear and demonſtrative, but alſo a Method vaſtly more general, as being applicable to other Machines where a Circle is of no Conſideration, and to the Caſe of parallel as well as converging Lines of Direction. Inſtances of this were given in the Vectis Libra, and Axis in *Peritrochio*. The Trochlea it ſelf is no other than a Vectis. And the Caſe juſt now conſider'd, is no other than that of a Vectis of the firſt kind, in which the Fulciment is equally diſtant from each Point of Suſpenſion.

Fig. 19.



PROB.

PROB. V.

Two Powers acting with given Directions, ſuſtaining a Weight by a Trochlea; 'tis requir'd to find the Proportion of either of thoſe Powers to the Weight.

LET the Powers (*FIG.* XIX.) at A and B (or the Power at A, and ſome Obſtacle well fixed at B) ſuſtain the *Pondus* by the Rope AEDB. The Line of Direction of the *Pondus* CF paſſes thro' C, the Center of the Trochlea; and thoſe of the Powers, which are AE and DB being produced, concur in F. Then by *Cor*. II. *Theor*. III. Power A : to the expended Weight A :: Sine Angle CFD : Sine Angle EFD. But AE and BD being Tangents, 'twill eaſily be prov'd, that the Angles EFC and DFC are equal. So that the Power is to the Weight, as the Sine of ½ the Angle made by the Lines of Direction to the Sine of that Angle it ſelf. Q : E : I.

COR.

When the Lines of Direction EF, DF, are parallel, then EC and CD, the Perpendiculars to them, will lie in one Right Line. And this will be reducible to a Vectis of the ſecond Sort, where one Power is applied between the Fulciment and the other Power. Conſequently the

Power at A will be to the Weight, as the *Radius* to the Diameter; that is, as 1 : 2.

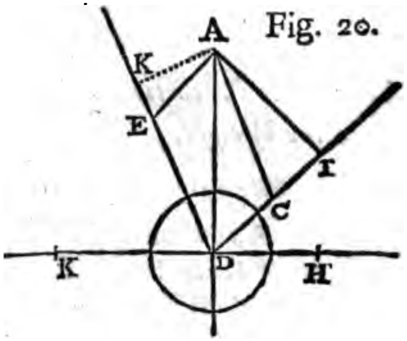


Fig. 20.

PROB. VI.

Two Powers acting with given Directions, ſuſtaining a Weight (without a Trochlea) 'tis requir'd to find what Proportion they bear to one another.

LET the Powers (*FIG.* XX.) at I and K, whoſe Directions are KD, and ID, ſuſtain the *Pondus* D. Produce the Lines of Direction till they meet in the Center D. Compleat the Parallelogram AEDC. Then by *Cor.* II. *Theor.* III. the Power at K, is to the Power at I :: as Sine Angle CAD : Sine EDA :: AC : CD, or AE. or which is all one, they are as AI : AK; the Lines AK, and AI being Perpendiculars from A to the Lines of Direction.

COR.

From hence one may eaſily compute the Powers of theſe Rhombs, which the fam'd *Borelli* conſiders, in order to the Eſtimation of the Forces of the Muſcles; and that, whether ſingle or combin'd together in Chains, according to all the Varieties mention'd in ſeveral Propoſitions in his excellent Book, *De Motu Animalium*. For the Proportion of thePowers not only to one onother, but alſo to the Gravity of the Weight, is eaſily deduced from that Third General *Theorem* that has been ſo often quoted already upon theſe Occaſions.

As for the Powers of the Wedge and the Screw, tho' they are not ſo immediately and directly explain'd, from the general Principle of the Compoſition of Forces, as thoſe others that we have conſider'd; yet they are included in it, being to be reduced to ſome one Head or other that is immediately demonſtrated from thence. Thus ſome People have referr'd both theſe Faculties to that of the enclin'd Plane; while others (whatever they do with the Screw) have reduced the Wedge to a double Vectis. But I am ſatisfied I can't do better in this Matter, than to give Mr. *Newton*'s own Account of them, and refer the Explication of the Wedge to that of the Body preſſing upon Two enclin'd

Planes making an Angle with one another, and make the Screw no other than a Wedge impell'd by a Vectis. The celebrated Dr. *Wallis* deduces the Properties of theſe (as of the reſt) from his general *Prop.* which is *Prop.* 5. *Chap.* 2. of his Tract, *De Motu.* But from which ſoever of theſe Principles a Man wou'd explain the Powers of theſe Machines, he will come to the ſame Concluſions; *viz.* For the Wedge; that when the impelling Force is to the Reſiſtance of the Obſtacle, as the Thickneſs of the Wedge, to the Height of the ſame: And for the Screw; that when the Force that turns the Screw is to the Reſiſtance of the Obſtacle, as the Interval of the Two immediately ſucceeding Turnings of the Spiral, to the Circumference deſcribed by the Power in one Converſion of the Screw: I ſay, that when it is ſo, the Powers (in theſe Machines) will be equivalent to the Reſiſtances of the Obſtacles, or (if one may ſo expreſs it) there will be an *Equilibrium* of Forces and Reſiſtances. Or, as our Great Authour expreſſes it, when the Force by which the Wedge urges the two Sides of the cleft Bodies is to the Force of the Mallet upon the Wedge, as the Progreſs of the Wedge according to the Determination of the Force impreſs'd upon it, to the Velocity with which the Parts of the Body give way to the Wedge in Lines perpendicular to the Sides of the Wedge; or when the Force of the Screw to preſs the Body, is to the Force of the Hand that turns it round, as the circular Velocity of the Handle, to the progreſſive Velocity of the Screw towards the compreſs'd Body; then there will be an *Equilibrium*, or the contrary Forces and Reſiſtances will ſuſtain one another.

But this holds true in all Machines whatſoever, that any Agent and Patient, Force and Reſiſtance, or contrary Force and Force, are then equivalent to, and will ſuſtain one another, when they are reciprocally as their reſpective Velocities, eſtimated according to the proper Directions and Determinations of thoſe Forces. For the Action of any Agent being eſtimated by its Force and Velocity conjunctly, and the Re-action of the Patient or reſiſting Body, eſtimated alſo by the Velocities and the Forces of Reſiſtance in the ſeveral Parts (ariſing from their mutual Attrition, Cohæſion, Weight or Acceleration) then Action and Re-action ſhall be equal to one another throughout all the Machines imaginable; or, in other Words, there will be an *Equilibrium*. The whole Buſineſs of *Mechanicks* may therefore be brought into a very narrow Compaſs, and the Powers of all Inſtruments whatſoever are included in this one general Analogy that follows. Let *p. r.* expreſs any Powers, moving Forces, or Forces and Reſiſtances; let the Velocity of *p.* be expreſs'd by m, and that of r. by n. Then if $p : r :: n : m$, the contrary Forces will ſuſtain one another, becauſe (upon this Suppoſition) $p \times m = r \times n$. Therefore if $p \times m > r \times n$; that is, if $\frac{r}{p} > \frac{m}{n}$; then the Force p. ſhall overcome the Force or Reſiſtance r. Hence in any Machine the Problem may be ſolved, *To move a given Weight (or to overcome a given Reſiſtance) with a given Force.*

And this one Application alone were ſufficient to ſhew, how true and certain, how general and extenſive that Third *Law of Motion* is, which tells us, that Re-action is ever equal to Action, *&c.* But this is but one Inſtance of the Uſe of it neither; there are ſeveral others beſides of no leſs Conſequence and Importance than this, which the Great Author himſelf has conſider'd, and his, and ſeveral other Examples relating to thoſe Matters, have been very well and largely explain'd by Mr. *Keil*; upon which Account I ſhall ſay the leſs to them. The ſame ingenious Perſon laſt mention'd has alſo in a late Book (his *Introduction to the true Phyſicks*) made uſe of the Second *Law of Motion* as a Principle to eſtabliſh ſome important Theorems upon, relating to Accelerated Motion; and the ſame Law will be of as great Uſe in demonſtrating all the common Theorems about equable Motions. But we are now to proceed with the *Corollaries* of thoſe *Laws.*

Second COROLLARY *to the Laws of Motion.*

The Quantity of Motion, which ariſes by taking the Sum of the Motion towards the ſame Part, or the Difference of thoſe made towards contrary Parts, is not changed by the Action of Bodies one upon another.

THAT is, if Two Bodies (moving either towards the ſame or contrary Parts) ſtrike each other, the Sum of the Motions made towards the ſame Part, and the Difference of thoſe made towards contrary Parts, will be the ſame after the Stroke that they were before. Let B, *b* be Two Bodies, V= the Velocity of B, and $v =$ the Velocity of *b*, S= the Sum of the Motions (made towards the ſame Part) D= the Difference of the Motions made towards contrary Parts. 'Tis plain, before theſe Bodies ſtrike one another, that the Sum of the Motions towards the ſame Part=S, is= $B\times V+b\times v$, (from that common Principle that the *Momentum* or Quantity of Motion of any Body is=to the Parallelogram under the Quantity of Matter it contains and its Velocity) and the Difference of the Motions towards contrary Parts=D is=$B\times V-b\times v$. Suppoſe now the Bodies B and b to ſtrike one another, and let the Action of B upon b be put $= m$, the Re-action of b upon $B=n$, then after the Colliſion the Motion of $B=B\times V\pm n$, and the Motion of $b=b\times v\pm m$, and conſequently the Sum or Difference ($=S$ or D) is $=\overline{B\times V\pm n}\pm\overline{b\times v\mp m}$, in which Expreſſion the Quantities m and n muſt be affected with different Signs; that is, if m be poſitive, n will be negative, and if n be poſitive, m will be negative, becauſe the Directions of the Action and Re-action are contrary. But (by the Third *Law of Motion*) Action is ever equal to Re-action; that is, the Quantity $m=-n$, or $n=-m$. Conſequently, ſubſtituting $-n$ in the room of m, and $-m$ in the room of n, they will deſtroy one another, and there will remain $B\times V+b\times v=S$, and $M\times V-m\times v=D$, the ſame Values as before. So that the Sum or Difference of the Motions is the ſame after the Colliſion that it was before.

From hence the Laws of the Colliſion and Motion of Bodies may be eaſily and univerſally computed.

Firſt, In perfectly hard Bodies; in order to which it's neceſſary that we premiſe this

LEMMA.

LEMMA.

If a hard Body ſtrikes directly upon another, whether that upon which the Stroke is made be quieſcent, or moves more ſlowly towards the ſame Quarter, or, Laſtly, towards the contrary Part (with a leſs Motion) after the Stroke they ſhall both move with the ſame Degree of Velocity.

LET the Body B ſtrike directly upon *b*. Then (by the Second *Law of Motion*) the Body *b* ſhall move according to the Direction of the Force impreſs'd upon it by the Body B; the Action and Influence of all circumjacent Bodies being ſuppoſed to be remov'd. But *b* cannot move ſlower than B, becauſe of B following it; nor can it move faſter, ſince (by the Hypotheſis) we remove all other Cauſes of its Motion, beſides the impelling Body B. Therefore the Bodies B and *b* ſhall move both with the ſame Velocity after their Colliſion.

From hence therefore to find true the Value of that common Velocity, with which the Body ſhall move after the Stroke: Let this common Velocity be put $=z$. Then $S=B\times V+b\times v$, and $D=B\times V-b\times v$, before the Stroke; and alſo (by *Cor.* II. to the *Laws of Motion*) the Values of S and D continue the ſame before and after the

Stroke. But after the Stroke $S=B\times z+b\times z$, and $D=B\times z-b\times z$. Therefore $\frac{B\times V+b\times v}{B+b}=z$, the common Velocity for the Sum of the Motions made towards the ſame Part; and $\frac{B\times V-b\times v}{B+b}=z$, the common Velocity for the Difference of the Motions made towards contrary Parts.

Hence if the Body b were quieſcent, then $v=0$, and ſo $z=\frac{B\times V}{B+b}$, that is, $V:z::B+b:B$; the Velocity before the Stroke is to that after it, as the Sum of the Two Bodies to the moving Body. Therefore if $B=b$, then $V:z::2:1$; that is, if the Bodies are equal, the Velocity after the Stroke will be half the Velocity of the Body, that was ſuppoſed in Motion, before the Stroke.

If v and V be both of ſome real Value, and alſo if $B=b$, then (for the Caſe of the Sum of Motions towards the ſame Part) we have $\frac{BV+Bv}{2B}=z$, or $z=\frac{V+v}{2}$; that is, the Velocity after the Stroke is ½ the Sum of the Velocities before the Stroke. For the Caſe of the Difference of Motions towards contrary Parts, we have $\frac{BV-Bv}{2B}=z$, or $z=\frac{V-v}{2}$, in which Caſe the Velocity after the Stroke is ½ the Difference of the Velocities before it.

But univerſally, let the the Bodies be in any *Ratio* whatſoever, the Value of z will be determin'd. Suppoſe $B : b :: p : q$. then $b = \frac{B\times q}{p}$, which Value ſubſtituted in the Equation above, will give the Value of z in this general Suppoſition. For then we ſhall find $z = \frac{pBV + qBv}{pB + qB}$ (for the Caſe of the Sum of Motions towards the ſame Part) and $z = \frac{pBV - qBv}{pB + qB}$ (for the Caſe of the Differences of Motions towards contrary Parts.) And if we ſuppoſe b quieſcent (as before) and conſequently $v = o$, then $z = \frac{pBV}{pB + qB}$, and $V : z :: p + q : p$.

In the Caſe of the Difference of Motions towards contrary Parts, if we put $z = o$; that is, if the Bodies, after their mutual Colliſion, have no Velocity at all, or the contrary Motions have deſtroy'd one another, then $\frac{pBV - qBv}{pB + qB} = o$, and $p\times V = q\times v$, or (becauſe of $p : q :: B : b$) $B\times V = b\times v$; that is, $B : b :: v : V$. So that the Bodies being reciprocally as their Velocities (which makes the contrary *Momenta* equal) after the Stroke they will loſe all their Motion.

And having thus determin'd what the Velocity muſt be after the Colliſion of any hard Bodies, there will be no Difficulty in finding the

Momenta, or Quantities of Motion in each of the ſtriking Bodies, after they have thus ſtruck one another; for there is no more to be done, but to multiply the common Velocity into the Magnitude of each Body.

And from hence we may eaſily determine the Magnitudes of the Strokes made by the Colliſion of one hard Body with another, and ſo arrive at thoſe *Theorems* which the celebrated Dr. *Wallis* has given us relating to this Matter, in the *Chap.* of Percuſſion, which is the 11th of his admirable Tract, *De Motu.* For (according to the Accounts of that excellent Perſon) the Magnitude of any Stroke is equivalent to twice the *Momentum*, loſt from the ſtronger of the Two Bodies that make the Shock (that is, ſuppoſing one to have more Force than the other.) But by the Rules above deliver'd we can find univerſally what the *Momentum* of any hard Body is after it has ſtruck another, and the *Momentum* before the Stroke being known likewiſe; 'tis plain, that we can find the Difference between the latter and the former of theſe *Momenta*; and twice that Difference is determin'd (*Prop.* 5. of the foremention'd *Chap.*) to be equivalent to the Magnitude of the Stroke. For the Doctor brings into the Account of the Stroke, not only what the ſtriking Body loſes but alſo what the Body that is ſtruck receives: Both theſe (he ſays) are the Effects of the Stroke, and conſequently the Magnitude of it is to be eſtimated from both together.

Here therefore I will from this Principle deduce Two of his moſt general *Theorems*, from whence the Reader may very eaſily draw the reſt himſelf.

Uſing the Symbols as before; let the Bodies B. *b*. whether equal or unequal, move with any Velocities towards the ſame Parts; and let the Body *b*. go before, and the Body B. come after with a Velocity V. greater than *v*. the Velocity of the former. The common Velocity after the Stroke (*viz.* z) $=\frac{BV+bv}{B+b}$; therefore the *Momentum* of B after the Stroke, is $=\frac{BBV+Bbv}{B+b}$; but its *Momentum* before the Stroke was $=BV$, therefore the Body B has loſt the *Momentum* $BV-\frac{\overline{BBV+Bbv}}{B+b}=\frac{BbV-Bbv}{B+b}$. But the Body *b* has acquired juſt as much; for its *Momentum* before the Stroke was $=bv$, and afterwards 'tis $=\frac{BbV+bbv}{B+b}$; therefore it has gotten the *Momentum* $\frac{BbV+bbv}{B+b}-bv=\frac{BbV-Bbv}{B+b}$. Conſequently, $\frac{2BbV-2Bbv}{B+b}$ is $=$ the Magnitude of the Stroke. But 'tis evident, that $\frac{2BbV-2Bbv}{B+b}:BV-Bv::2b:B+b$; that is, the Magnitude of the Stroke, is to the *Momentum* of the following Body (B) carried with the Difference of the Velocities, as twice the antecedent Body (b) to the Aggregate of both the Bodies.

In like manner, if the Bodies mov'd towards contrary Parts, it will be found, that $\frac{2BbV+2Bbv}{B+b}$: $BV+Bv$ :: $2b : B+b$; that is, the Magnitude of the Stroke, wou'd be to the *Momentum* of either of the Bodies (*Ex. gr.* B) carried with the Sum of the Velocities, as twice the other Body (*viz.* b) to the Aggregate of both the Bodies.

And from theſe Two *Theorems* one may deduce any of the reſt, according to the Nature of the Suppoſition made.

COR. I.

Joining the Reſult of both theſe Analogies together, it follows, that $\frac{2BbV-2Bbv}{B+b}$: $\frac{2BbV+2Bbv}{B+b}$ :: $BV-Bv : BV+Bv$ :: $V-v$: $V+v$; that is, the Magnitude of the Stroke in the Caſe of Motions made towards the ſame Parts, is to the Magnitude of the Stroke in the Caſe of Motions towards contrary Parts, as the Difference of the Velocities in the former Suppoſition, to the Sum of the Velocitiy in the latter; that is, as the relative Velocities of the Bodies in both Caſes after the Colliſion.

COR. II.

In either Caſe conſider'd apart by it ſelf, 'tis plain, that whatever the Velocities are in themſelves, yet if the Difference, or the Sum of them continues ſtill the ſame, the Magnitude of the Stroke ſhall ſtill be the ſame alſo.

And therefore it follows, that the Magnitude of the Stroke is ever proportional to the Difference or the Sum of the Velocities; that is, to the relative Velocity of the Bodies, by which they come towards one another.

From the *Laws of Motion* in perfectly hard Bodies, we come to conſider.

2. In perfectly Elaſtick Bodies; and here 'tis neceſſary to premiſe this

LEMMA.

That if Two Bodies perfectly Elaſtick ſtrike one another, their relative Velocity continues the ſame after the Stroke, that it was before.

FOR the Nature of a Body perfectly Elaſtick is to reſtore it ſelf with a Force equal to that by which it was compreſs'd. But (in the Hypotheſis of the preſent *Lemma*) the Compreſſive Force is no other than the Force of the

Stroke, by which one of the Elaſtick Bodies ſhocks the other.

Now the Magnitude of the Stroke (by the Second *Cor.* foregoing) ariſes from the relative Velocity of the Bodies, and is proportional to it. Conſequently the compreſſive Force is proportional to that Velocity. But the reſtitutive Force being equal to the compreſſive, acting upon the ſame Bodies, ſhall produce a relative Velocity that bears the ſame Proportion to it, that the relative Velocity before the Stroke bore to the compreſſive Force; therefore it ſhall produce a relative Velocity equal to the former. And therefore the Body's after Colliſion ſhall depart from each other with the ſame Velocity that they came towards one another with before. Q: E: D.

From hence now to determine the Velocities of Elaſtick Bodies after the Stroke.

All the other Symbols ſtanding as before, let the Velocity of the Body B after the Colliſion be put $=z$. When the Bodies B. *b.* move towards the ſame Parts, then their relative Velocity is $=V-v$; and when towards contrary Parts, 'tis $=V+v$. Therefore (by the foregoing *Lemma*) in the former Caſe, the Velocity of the Body *b.* after the Stroke ſhall $=z+V-v$; and in the latter Caſe it ſhall $=V+v+z$. But (by *Cor.* II. to the *Laws of Motion*) the Sums of the Motions towards the ſame Parts, or the Differences towards contrary Parts are equal, both before and after the Stroke. Therefore $BV+bv=Bz+bz+bV-bv$, in one Caſe, and $BV-bv=Bz+bV+bv+bz$, in the other.

Therefeee $z=\frac{BV+2bv-bV}{B+b}$, in one Caſe, and $z=\frac{BV-2bv-bV}{B+b}$, in the other. And putting the Velocity of the Body *b* after the Stroke $=y$; we have $y\ (=z+V-v)=\frac{2BV+bv-Bv}{B+b}$, in one Caſe; and $y\ (=z+V+v)=\frac{2BV+Bv-bv}{B+b}$, in the other Caſe. So that we have theſe 2 general Analogies, *viz.* $z:y::BV+2bv-bV:2BV+bv-Bv$, when the Bodies B. and *b*. move towards the ſame Parts; and $z:y::BV-2bv-bV:2BV+Bv-bv$, when the Motions are made towards contrary Parts. And theſe Values of *z* and *y* do giye us Dr. *Wallis*'s 9th and 10th *Props. Chap.* 13. *Mcchan.*

And from theſe general Expreſſions thus found, all the particular Laws are eaſily determin'd.

Ex. gr. Suppoſe the Body *b* were quieſcent; then $v=o$, and conſequently $z=\frac{BV-bV}{B+b}$, and $\frac{BV-bV}{B+b}:V::B-b:B+b$; that is, the Velocity of the moving Body B, after the Stroke, is to the Velocity it had before, as the Difference of the Two Bodies to the Sum of them. Again, (uſing the ſame Suppoſition) $y=\frac{2BV}{B+b}$, and $\frac{2BV}{B+b}$: V :: 2 B : B+b; that is, the Velocity of the quieſcent Body *b* after the Stroke is to the Velocity of the ſtriking Body, B before the Stroke, as twice the ſtriking Body, to the Sum of both Bodies. If B=*b*, then *b*, ſuppoſed quieſcent as before, *z* will = *o*; that is, the Body B after the Stroke will loſe all its Motion. And all this Dr. *Wallis* ſhews *Prop.* 8. *Chap.* 13. concerning Elaſtick Force and Reflection.

Suppoſe B=*b*, and that the Bodies mov'd towards contrary Parts. Then $z=-\frac{2bv}{B+b}$, and $y=\frac{+2BV}{B+b}$; but becauſe B=*b*, therefore $\frac{-2bv}{B+b}=\frac{-2Bv}{B+b}$. So that after the Stroke, the Bodies ſhall mutually interchange their Velocities, the Body B moving with the Velocity of *b* before the Stroke, and the Body *b* with the Velocity of B before the Stroke. And that they ſhall move towards contrary Parts is manifeſtly enough ſignified by the Difference of the Signs, for it is +V, and —*v*. And this is *Prop.* 7. of the foremention'd *Chap.*

Suppoſe again B = *b*, and that they mov'd towards the ſame Parts, the Body *b* going before with a Velocity = *v*, and the Body B coming after with a greater Velocity, *viz.* V. Then $z = \frac{+2bv}{B+b}$, and $y = \frac{+2BV}{B+b}$. Therefore after the Stroke, the Bodies ſhall move with the Velocities they had before the Stroke, alternately taken, as it was in the Caſe foregoing, only with this Difference, that there they mov'd towards contrary Parts, and here towards the ſame Parts, as is plain from the Identity of the of the Signs. And this is *Prop.* 6.

If $B=b$, but the Body b be ſuppos'd quieſcent; then $z=o$, (for in this Caſe $v=o$, and $bv=o$, and becauſe $b=B$, then $BV-bV=o$, and ſo the whole Value of $z=o$) but $y = \frac{2BV}{B+b} = \frac{2BV}{2B} = V$. So that after the Stroke the Body B at firſt in Motion ſhall quieſce, and the Body b at firſt quieſcent ſhall move, with a Velocity $=V$, the Velocity of B before the Stroke. And this is *Prop.* 5.

If B were $=b$, and alſo $V=v$, and the Bodies mov'd towards contrary Parts, then 'tis plain, that $z=-v$, and $y=+v$; that is, the Velocity of each Body after the Stroke is the very ſame it had before, or they are reflected each with thoſe Velocities with which they meet together. Here alſo the Difference of the Signs diſcovers, that the reflected Motions are made towards contrary Parts; and the Nature of an Elaſtick Body (which ever reſtores it ſelf in the ſame Direction according to which it was compreſt) as plainly ſhews, that the Bodies will move in the very ſame Right Lines after the Stroke, that they did before.

Again, if neither $B=b$, nor $V=v$, yet if the the Bodies be reciprocally as their Velocities; that is, if $B:b::v:V$; in this Caſe alſo after the Stroke they ſhall move towards contrary Parts with the ſame Velocities they had before. For ſince (by the Suppoſition) $B\times V=b\times v$, therefore (the Motions being made towards contrary Parts) z ſhall $=\frac{-bv+bV}{B+b}$, and y ſhall $=\frac{+BV+Bv}{B+b}$. But alſo $\frac{-bv-bV}{B+b}=-V$ and $\frac{+BV+Bv}{B+}$ $=+v$; therefore the Bodies ſhall move with the ſame Velocities reſpectively towards contrary Parts after their Colliſion. And this is *Prop.* 3. of the forementioned *Chap.*

Having now conſider'd theſe particular Laws of Motion, I proceed to

Third

Third COROLLARY *to the general Laws of Motion or Nature.*

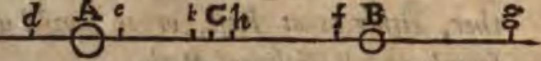
The common Center of Gravity alters not its State of Motion or Reſt by the Actions of Bodies upon one another; and therefore (ſetting aſide all external Actions and Impediments) the common Center of Gravity of all Bodies acting mutually upon one another, either is at Reſt, or is mov'd uniformly in a Rectilineal Path.

THE common Center of Gravity of Two Bodies, is the Center or fix'd Point of an imaginary *Libra*, about which the Bodies are in *Equilibrio*. This *Libra* is a Line conceiv'd to paſs thro' the Center of Gravity of each Body or to be continued from one Body's Center of Gravity to that of the other. Then the Fulciment or fix'd Point (found by taking the Diſtances of the Centers of Gravity of the Bodies from that Point, reciprocally proportional to the Magnitudes of the Bodies themſelves) is what we call the common Center of Gravity of the Two Bodies. So likewiſe we may make an Eſtimate of the common Center of Gravity of any Number or Syſtem of Bodies, according to this Principle.

The Demonſtration of this *Coroll.* will be clear, upon the making out of theſe Two Proportions.

1. That Bodies moving uniformly, either towards the ſame, or towards contrary Parts; before they meet and ſtrike one another, their common Center of Gravity will either quieſce, or move uniformly in a Right Line.

Fig. 21.



Let the Bodies (*FIG.* XXI.) be A. B. their common Center of Gravity C. the Diſtances of their Center of Gravity from that Point, AC and CB reſpectively. And let the ſmall Letters denote different Places of the ſame Bodies, and of their common Center of Gravity.

From the Nature of the common Center of Gravity, 'tis plain that $A : B :: BC : AC$, from whence $A \times AC = B \times BC$. Now if the Bodies mov'd into the Places d. g. or e. f. and it were ſo that $A : B :: gC : dC$, or $A : B :: fC : eC$; then the Motions with reſpect to the ſame Point C, wou'd ſtill be equal, becauſe $A \times dC = B \times gC$, or $A \times eC = B \times fC$) and the common Center of Gravity C wou'd quieſce. So that in equal Motions towards contrary Parts, whether the

Bodies go from, or come towards one another, the common Center of Gravity is ſtill at reſt.

But if the Motions were ſo adjuſted to each other, that $A : B > gC : dC$, or $A : B > fC : eC$, that is, $A \times dC > B \times gC$, or $A \times eC > B \times fC$; then ſome Point as k between e or d and C, may be taken ſo, that $A \times dk$ may $= B \times gk$; and then 'tis plain, that the Point k is the common Center of Gravity. Therefore in this Caſe, the common Center of Gravity has mov'd from the Point C to the Point k. So likewiſe it might be ſhown, that if $A \times dC < B \times gC$, or $A \times eC < B \times fC$, then the common Center of Gravity wou'd be in ſome Point as h between g or f and C, and ſo to have mov'd the Space Ch. So that 'tis moſt clear, that in unequal Motions towards contrary Parts, the common Center of Gravity moves in the ſame Right Line together with the Bodies themſelves. Again, if the Bodies move towards the ſame Parts with any Motions whatſoever, it is manifeſt, that the common Center of Gravity, muſt move alſo. Let the Bodies A and B whoſe common Center of Gravity is C, move from the the Places A and B, into the Places e and g; I ſay, the common Center of Gravity cannot be now in the Point C. For if it were, then it wou'd be $A : B :: gC : eC$, the Bodies being in e and g. But it is $A : B :: BC : AC$, the Bodies being in A and B. Therefore it wou'd be $gC : eC :: BC : AC$, that is $gC : BC :: eC : AC$, which is impoſſible. Therefore the common Center of Gravity is not in C, but into ſome Point as h taken ſo, that it may be $gh : eh :: BC : AC$, or $gh : BC :: eh : AC$. And that it muſt neceſſarily be in ſome Point, as h between C and B, is too plain to need any Proof. And further, 'tis evident enough, that the Motion of the common Center of Gravity (whenever it moves, in any of theſe Caſes) is uniform, or that it deſcribes equal Spaces in equal Times. For the Motion of the Bodies is uniform (by the Suppoſition) and the common Center of Gravity divides the Diſtance between the Bodies ever in the ſame *Ratio*; therefore as the Spaces deſcribed by the Bodies themſelves in equal Times, will be equal, ſo alſo will thoſe deſcrib'd by the common Center of Gravity.

Hitherto we have conſider'd the Bodies only as moving in one and the ſame Right Line. But if they move in different Right Lines (either parallel, or enclin'd to one another) the Motion of the common Center of Gravity may ſtill be ſhewn to be perform'd in a Right Line, and to be uniform; for the Demonſtration of which I refer the Reader to Mr. *Keil's* forementioned Book. So alſo if the Bodies mov'd in different Planes, and it were ever ſo great a Number of Bodies, the Motion of whoſe common Center of Gravity was conſider'd.

2. The next Thing to be clear'd, is, That when Two Bodies move either towards the ſame or towards contrary Parts, the Sum or Difference of their Motions is the ſame as if they were both carried with the Velocity of their common Center of Gravity.

This *Prop.* may be eaſily demonſtrated, but I will at preſent give an Inveſtigation of it. And the doing it in one Caſe alone, when the Bodies move towards the ſame Parts, will be Direction enough to the induſtrious Reader to do it in the other Caſe himſelf. In (*FIG.* XXI.) Let the Bodies A and B move from the Places A and B into the Places e and g, and their common Center of Gravity from C into h, at the ſame time. Let $AB=b$, $AC=c$. Then $BC=b-c$. Let $Ch=n$. $Ae=d$. Then $Bh=b-c-n$. and $eC=c-d$. and $eh=c-d+n$.

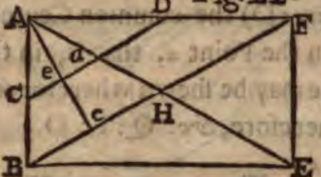
When the Bodies are in e and g, the Value of gh will be found $=\frac{cb-cc-db+dc+nb-nc}{c}$, from the Nature of the common Center of Gravity. Therefore $Bg\ (=gh-Bh)=\frac{dc+nb-db}{c}$. But the Sum of the Motions of the Bodies is $=A\times Ae+B\times Bg$, that is, $A\times d+\frac{B\times dc+B\times nb-B\times db}{c}$. And the Motion of both carried with the Velocity of the common Center of Gravity is $=A\times Ch+B\times Ch$, that is, $A\times n+B\times n$. And (multipling both Parts by c) we ſhall find $A\times nc+B\times nc=A\times dc+$

$B \times dc + B \times nb - B \times db$ (becauſe $A \times c = B \times b - B \times c$, from the Nature of the common Center of Gravity, and conſequently $A \times nc = B \times nb - B \times nc$, and $A \times dc = B \times db - B \times dc$.) The Reſult therefore is that the Motions are the ſame, as if both Bodies were carried with the Velocity of the common Center of Gravity. Q: E: I.

And now from both theſe *Props.* together ariſes a Demonſtration of the Truth of this Third *Cor.* For (by the Second *Cor.* the Sum of the Motions towards the ſame Quarter remains the ſame, after the Bodies have ſtruck one another, that it was before. But it has been ſhewn, that the Sum of the Motions either before or after the Stroke is the ſame as if the Bodies had been carried with the Velocity of the common Center of Gravity. From whence it may certainly be inferr'd, that the Velocity of the common Center of Gravity, is the ſame before as after the Colliſion of the Bodies. But if the common Center of Gravity moves at all before the Bodies meet and ſtrike, it moves uniformly in a right Line, therefore after the Bodies have met and ſtruck, it will move uniformly in a right Line alſo: As on the other Hand, if it be at reſt before the Stroke it will eaſily be ſhewn to be at reſt afterwards. Therefore the common Center of Gravity does not change its State of Reſt or uniform Motion in a rectilineal Direction by any Action of Bodies upon one another. Q: E: D.

And thus much for the Illuſtration of the *Coroll.* But before I paſs to the next, (being now upon this Head) I'll add Two or Three *Props.* relating to the Motion of the common Center of Gravity.

Fig. 22.



THEOR. I.

If Two Bodies equal in Magnitude, move in the Sides of a Parallelogram with Velocities proportional to thoſe Sides reſpectively; their common Center of Gravity ſhall ſtill be found in the Diagonal.

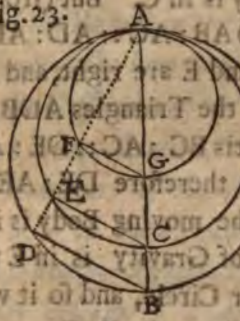
LET the Bodies (*FIG.* XXII.) move in the Lines FA, BA, from the Points F and B towards A. Draw the Diagonals AE, BF, cutting each other in H, and any other Line as CD parallel to the Diagonal BF, interſecting the Diagonal AE in the Point *a.* When the Bodies are in the Points B, F, 'tis plain, that their common Center of Gravity is in H; for the Bodies are equal by the Hypotheſis, and BH is = HF (from the Elements of the common Geometry.) Likewiſe becauſe the Velocities of deſcribing the Sides of the Parallelogram are ſuppos'd proportional to thoſe Sides reſpectively, therefore (CD being parallel to BF) when one of the Bodies is in D, the other will be in C, for CB : DF :: AB : AF ; conſequently BC and DF will be diſcrib'd in the ſame time. Therefore (becauſe $Ca=aD$) the common Center of Gravity will be in the Point a, that is, in the Diagonal. The ſame may be ſhewn whereſoever the Bodies are. Therefore, *&c.* Q: E: D.

THEOR. II.

If the Bodies were anequal in Magnitude; the Velocities with which they move being as the Sides of the Parallelogrom reſpectively; the Path of the common Center of Gravity (which will not be the Diagonal) is eaſily determin'd.

LET the *Ratio* of the Magnitude of the Bodies be what it will; then from A (*FIG.* XXII.) draw the Line Ac, to cut the Diagonal BF in the Point c in ſuch a Manner, that the Segments Bc, and cF may be reciprocally as the Magnitudes of the Bodies plac'd in B and F. 'Tis evident, that the common Center of Gravity ſhall deſcribe the Path Ac, while the Bodies themſelves deſcribe the Sides of the Parallelogram, BA and FA. For any Line as CD parallel to BF, is cut by the Line A *c* ſo, that the Segment C *c*, *c* D are ever in the ſame *Ratio* of B*c* and *c*F. Therefore, *&c.* Q : E : D.

Fig. 23.



THEOR. III.

If Two Circles touch one another internally, and one Body be at reſt in the Point of Contact, while another moves in the Periphery of the outer Circle; then if the Sum of the Two Bodies be to the moving Body, as the Diameter of the outer Circle, to the Diameter of the innermoſt, the common Center of Gravity ſhall deſcribe the Periphery of the innermoſt Circle.

LET (*FIG.* XXIII.) the quieſcent Body be fix'd at A, the Point of Contact of the Two Circles, and let the other move from B to D in the outer and greater Circle. And let theſe Bodies be called A and B reſpectively. Then by the Hypotheſis 'tis A+B : B :: AB : AC. Therefore (by Diviſion) A : B :: BC : AC; conſequently when the moving Body is in B, the common Center of Gravity is in C. But (from the Nature of the Circle) AB : AC :: AD : AE (becauſe the Angles at D and E are right, and that at A is common, and to the Triangles ADB and AEC are Similar) alſo 'tis BC : AC :: DE : AE. But BC : AC :: A : B, therefore DE : AE :: A : B. Therefore when the moving Body is in D, the common Center of Gravity is in E the Periphery of the inner Circle, and ſo it will be always found in the ſame. Q: E: D.

THEOR. IV.

*If Three Circles touch'd one another in the ſame Point A, (*FIG. XXIII.*) it might be ſhewn, that if the Body A mov'd in the Periphery AFG, and the Body B in the Periphery ADB, and the Sum of the Two Bodies were to the Body B, as the Difference of the Diameter of the outer and inner Circles, to the Difference of the Diameters of the middle and inner Circles. Alſo if the Velocities of the Bodies were to one another directly as the Peripheries which they deſcribe; I ſay then, that the common Center of Gravity ſhall deſcribe the Periphery of the middle Circle,* viz. *AEG.*

FOR if the Velocities be as the Peripheries, they are alſo as any Two ſimilar Arches FG and DB cut off by any right Line AFD drawn at Pleaſure; therefore in the ſame Time that one Body deſcribes the Arch BD, the other will deſcribe the Arch FG; that is, when one is in D, the other will be in F in the ſame right Line AFD. Alſo becauſe (by the Hypotheſis) 'tis $A+B : B :: BG : GC$, by diviſion 'tis $A : B :: BC : GC$. But (from the Nature of the Circle and the Similarity of the Triangles) 'tis $BC : GC :: DE : EF$, therefore $A : B :: DE : EF$.

And therefore the common Center of Gravity is in the Point E of the middle Circle.

COR.

Hence if the Velocities of the Body were reciprocally proportional to their Magnitudes, then AB the Diameter of the outer Circle, wou'd be divided Harmonically, in the Points, A, G, C, and B, and *Vice Verſâ.* If the Lines BC, GC, AB, AG be in muſical Proportion, then the Velocities of the Bodies are reciprocally proportional to their Magnitudes. Let the Velocity of the Body B be put $= V$, and that of the Body A be put $= v$. Then (by the Hypotheſis of the *Theorem*) 'tis $V : v :: AB : AG$, for the Velocities which are as the Peripheries, are alſo as the Diameters. But (becauſe of the common Center of Gravity) 'tis $A : B :: BC : GC$. Therefore if it be $V : v :: A : B$, (that is, if the Magnitudes of the Bodies are reciprocally as their Velocities) then it follows, that $AB : AG :: BC : GC$, and conſequently $AB \times GC = AG \times BC$, which is the Property of a Line cut in muſical proportion. The Line AB therefore will be divided Harmonically.

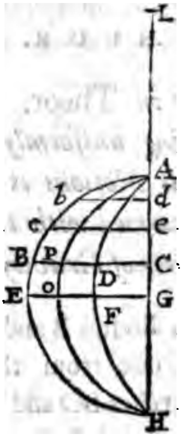
THEOR. V.

Suppoſing all as in Theor. 4. *I ſay, that Two Bodies moving uniformly in Circles, the Sum of their Motions is the ſame, as if both were carried with the Velocity of the common Center of Gravity.*

WHile the Bodies B and A (*FIG.* XXIII.) Setting out from the Points B and G, deſcribe the Arches BD and GF; the common Center of Gravity deſcribes the Arch CE, as has been already demonſtrated. The Sum of the Motions of both therefore, carried each with their own Velocities, is $= B \times BD + A \times FG$; and the Sum of the Motions, if both were carried with the Velocity of the common Center of Gravity, wou'd $= A \times EC + B \times EC$. Now I ſay, that $A \times EC + B \times EC = B \times BD + A \times FG$. For (from the Nature of the common Center of Gravity) 'tis $A : B :: BC : GC$, or (becauſe $BC = AB - AC$, and $GC = AC - AG$) $A : B :: AB - AC : AC - AG$. From whence $A \times AC - A \times AG = B \times AB - B \times AC$; and $A \times AC + B \times AC = B \times AB + A \times AG$. But becauſe the Arches DB, EC, FG are ſimilar, they are porportional to their reſpective Diameters. Therefore $A \times EC + B \times EC = B \times DB + A \times FG$ Q: E: D.

THEOR.

Fig. 24.



THEOR. VI.

If a Body moves in the Curve of a Conick Section, *whilſt another moves in the Ax of the ſame, and their Motions be adjuſted ſo, that while one deſcribes any Portion of the Curve, the other deſcribes the correſpondent Abſciſs, their common Center of Gravity ſhall always deſcribe one of the* Conick Sections.

CASE I.

SUppoſe the Curve ABE (*FIG.* XXIV.) were a Circle, whoſe *Radius* is AG. Let one Body be imagin'd to deſcribe AB or AG in the Ax, while the other deſcribes AB or AE in the Circumference; and in the *Radius* EG, let ſome Point as F be taken, dividing the Line EG, in the *Ratio* of the Magnitudes of the Bodies. I ſay, that an Ellipſe, whoſe Tranſverſe Ax is equal to the Diameter of the Circle, and whoſe Semi-conjugate is equal to FG (that is, ſuppoſing EGA a Right Angle) ſhall be the Path deſcrib'd by the common Center of Gravity. For, conceive an Ellipſe paſſing thro' the Points A, F, H; and draw the Ordinate BC, *c e*, *&c.* cutting an Ellipſe in D, *n*, *&c.* Then (from the Nature of the Circle $EG^q = AG \times GH$, and $BC^q = AC \times CH$. And from the Nature of the Ellipſe, $FG^q : DC^q :: AG \times GH : AC \times CH$. From whence $EG^q : FG^q :: BC^q : DC^q$; and EG : FG :: BC : DC, and (by Diviſion) EF : FG :: BD : DC, and ſo the Curve of the Ellipſe will divide all the Ordinates of the Circle in the ſame ſtanding *Ratio* of EF to FG. Therefore if EF : FG :: as the Body in G : to the Body in E, then their common Center of Gravity will be in F; and when the Bodies are in B and C, 'twill be in D; when they are in *c* and *e*, 'twill be in *n*. Therefore the common Center of Gravity will deſcribe the Curve of this Ellipſe. Q: E: D.

CASE II.

If the Curve OAH were an Ellipſe, having the Tranſverſe Ax AH; it might be ſhewn, that the common Center of Gravity would deſcribe another Ellipſe, as ADF, having the ſame Tranſverſe with the former. For (from the Nature of theſe Gurves) OF : FG :: *p*D : DC. So that a Body moving in the Ellipſe AOH, while another moves in the Ax; dividing the Semi-conjugate in the Point F, in the *Ratio* of the Magnitudes of the Bodies; the common Center of Gravity ſhall deſcribe the Ellipſe AFH.

CASE III.

If one Body deſcrib'd the Circle AEH, and another the Ellipſe AFH; then dividing the Line EF in the Point O, in the *Ratio* of the Magnitudes of the Bodies; it may be prov'd after the ſame Manner, that the common Center of Gravity ſhall deſcribe the Ellipſe AOH, paſſing thro' the Point O, and having the ſame Tranſverſe Ax AH. For it may be demonſtrated as above, that EF : FG :: BD : DC. And that OF : FG :: *p*D : DC. Therefore EF : OF :: BD : *p*D; and (by Diviſion) EO : OF :: B*p* : *p*D. So that if when the Bodies are in E and F, the common Center of Gravity be in the Point O; then when they are in B and D, it ſhall be in the Point *p*. Therefore the common Center of Gravity ſhall deſcribe the Ellipſe AOH.

CASE IV.

If the Curve ABE were a Parabola, whoſe Ax is AG, and Ordinate EG, it may be demonſtrated,

ſtrated, that the common Center of Gravity ſhall deſcribe another Parabola (as ADF, or ApO) having the ſame common Ax. For (from the Nature of theſe Curves) 'tis $EG^q : BC^q :: AG : AC$. And $FG^q : DC^q : AG\ DC$, therefore $EG^q : BC^q :: FG^q : DC^q$, and $EG : BC :: FG : DC$, and alternately $EG : FG :: BC : DC$, and (by Diviſion) $EF : FG :: BD : DC$, and ſo 'tis every where. Conſequently if EG be divided in F in the *Ratio* of the Magnitudes of the Bodies, the common Center of Gravity ſhall deſcribe the Parabola ADF, or (which is all one) if upon the Ax AG and with a Parameter $= \frac{FG^q}{AG}$ we deſcribe a Parabola, this Curve ſhall be the Path of the common Center of Gravity.

CASE V.

In like manner it might be ſhewn, that if the Three Curves ABE, ApO, ADF were Parabola's; the Bodies deſcribing the outer and the inner Parabola's, the common Center of Gravity wou'd deſcribe the middle one, *viz.* ApO; that is, ſuppoſing their Motions to be ſo adjuſted as was mention'd in the *Theorem*, and the Interval EF to be divided in the Point O, in the *Ratio* of the Magnitudes. For (in theſe Curves alſo) $EO : OF :: Bp : pD$.

CASE. VI.

And, Laſtly, If the Curve ABE were an Equilateral Hyperbola, having the Tranſverſe AL, and Ordinate EG; it may be demonſtrated, that the common Center of Gravity ſhall deſcribe another (but not an Equilateral) Hyperbola, having the ſame Tranſverſe AL with the former; or, which is all one, that ſuch on Hyperbola as this ſhall be the Track of the common Center of Gravity, as the Bodies move one in the Curve, and the other in the Ax, according to the Intent of the *Theorem.* For in the Equilateral Hyperbola $EG^q = LG \times AG$, and $BC^q = LC \times AC$; and in the other Hyperbola (from the *Conick* Elements) 'tis $FG^q : DC^q :: LG \times AG : LC \times AC$. So that in theſe Curves alſo 'twill be concluded, that $EF : FG :: BD : DC$; and from thence the Conſequence (ſo often repeated) with Reference to the common Center of Gravity, is plain and clear. And the ſame may be ſhewn here of Three Hyperbola's, as was before of Three Parabola's, or of Two Ellipſes and a Circle.

There are ſome of the *Mechanick* Curves, in which I cou'd apply this Matter yet farther; but I muſt obſerve juſt Bounds, and (as 'tis now time) leave this Subject.

Cor.

'Tis evident from theſe ſeveral Caſes, that the common Center of Gravity deſcribes a *Conick Section* of the ſame Family with that which the Bodies themſelves deſcribe; if at leaſt (as to the firſt *Caſe*) one may be allow'd to make the Circle and Ellipſe akin to one another; and ſay, that a Circle is an Ellipſis, whoſe Tranſverſe and Parameter are equal to one another.

Fourth Corollary *to the general Law of Motion.*

The Motions of Bodies (included in any given Space) are the ſame with reſpect to one another, whether that Space be at Reſt, or whether it moves uniformly in a Right Line, without any circular Motions.

THIS is eaſily demonſtrated from the Second *Corollary* to the *General Laws of Motion*; and particularly from the Deduction made from thence, about the relative Velocity being ever proportional to the Magnitude of the Stroke. If the relative Velocities continue ever the ſame (as they do, whether the Space, in which they are contain'd, moves, or is at reſt) then the Magnitudes of the Strokes which the Bodies make upon one another, are the ſame ſtill, and ſo their Motions are the ſame with reſpect to one another.

Fifth COROLLARY *to the general Laws of Motion.*

If Bodies are mov'd any how amongſt themſelves, and are urg'd by equal accelerating Forces, that act with parallel Directions, their Motions will proceed in all Reſpects, as if they were not influenc'd at all by thoſe accelerating Forces.

FOR thoſe Forces acting equally (with reſpect to the Quantities of the Bodies to be moved) and in parallel Directions alſo; they will (by the Second *general Law of Motion*) as to the Point of Velocity, equally move the ſeveral Bodies, and conſequently never alter their Motions and Poſitions with Reſpect to one another.

Having thus endeavour'd to make ſome Applications of thoſe moſt uſeful and comprehenſive *Laws of Nature* laid down by the *Great* and *Illuſtrious Author*; I ſhall go on to *his Doctrine* of *Centripetal Forces*, and explain ſome of the wonderful *Theorems* he has diſcover'd upon that Head.

The End of the Firſt Part.

OF THE Centripetal Forces, AND *Velocities of Bodies,* Moving in Curvilineal Figures.

PART II.

DEFINITION I.

Centripetal Force is that, by which a Body is drawn, or impell'd, or by any other Means (howſoever) made to tend to a certain Point, as a Center.

OF this kind is the Force of Gravity, by which a Body tends to the Center of the Earth; the Magnetick Force by which a Body tends to the Center of the Load-ſtone; and that Force (whatever it be) by which the Planets are continually with-held from going on in Right Lines, and forced to move in Curves.

DEFINITION II.

The abſolute Quantity of the Centripetal Force, is the Meaſure of it, greater or leſs, in Proportion to the Efficacy of the Cauſe, propagating it a Centro ad circumferentiam.

THus in one attractive Body, the Power of Attraction is ſtronger and greater than in another. The Magnetick Vertue is more vigorous and forcible in one Load-ſtone, than in another. And (in the greater Syſtems of Matter) that Force, by which the ſame Bodies, at the ſame Diſtance from the Centers of the Planets, is urged towards them, is ſtronger in one Planet than another. Thus (as we ſhall ſee afterwards) the Weight of the ſame Body at any Diſtance from the Center of the Sun is greater than its Weight at the ſame Diſtance from the Center of *Jupiter*; and this greater than that, at the ſame Diſtance from the Center of *Saturn*; and this greater than that, at the ſame Diſtance from the Center of the Earth.

DEFINITION III.

The accelerating Quantity of the Vis Centripeta, *is the Meaſure of it, with reſpect to the Velocity which it generates in a given Time.*

Ex. gr. The Vertue of the ſame Magnet is greater at a leſs Diſtance, and leſs at a greater Diſtance. The gravitating Force of Bodies, is greater in the Valleys and depreſs'd Places of the Earth, than upon the Tops of high Hills and Mountains; and is yet greater in theſe elevated Places, upon the Surface, than at greater Diſtances from it. But in equal Diſtances from the Earth, 'tis every where the ſame; for all deſcending Bodies, be they more heavy or light, greater or ſmaller, abating the Reſiſtance of the Air, are equally accelerated. The manifold Experiments made upon *Pendulums* have ſufficiently prov'd theſe Things to be true in Fact.

DEFINITION III.

The Quantitas Motrix *of the Centripetal Force, is the Meaſure of it, proportional to the Motion which it generates in any given Time.*

Such is more or greater Weight in a greater Body, and a leſs in a leſs Body; and greater

Weight in the ſame Body near the Earth, and leſs at a farther Removal upwards. This Force is the *Centripetentia*, or Propenſion of a Body towards a Center, or (as one may properly enough ſay) the *Pondus* of it; and 'tis always known (that is, the Magnitude of it is meaſured or found) by that of an equal and contrary Force, ſuch as is ſufficient to hinder the Deſcent of the Body.

EXPLICATION.

Theſe Quantities of the Centripetal Force Mr. *Newton* calls by theſe Names, and, for Diſtinctions ſake, conſiders them as referring to Bodies, to the Places of Bodies, and to a Center of Forces. Namely, the *Vis Motrix* relates to Bodies themſelves, as the Endeavour and Propenſion of the whole (compounded of the Propenſions of all the Parts) towards a Center. The *Vis Acceleratrix* reſpects the Place of a Body, as a certain Force or Efficacy (propagated from a Center, and diffuſed thro' all the circumjacent Regions) to move Bodies that are in thoſe Places. The *Vis Abſoluta* reſpects the Center it ſelf, as ſome way or other a Cauſe, without which the *Vires Motrices* are not propagated thro' the Regions about it. What the Cauſe and Origin of Centripetal Force is, is not neceſſary to be determin'd here; 'tis ſufficient that there is ſuch a Thing in Fact. As for Matters of pure Phyſical Conſideration (ſuch as are the Cauſes and Subjects of theſe Forces) 'twas Foreign to our Author's Purpoſe and Deſign to enter upon any Diſcuſſion of them in this Place. But here we may take Notice of an uſeful Compariſon between the *Vis Acceleratrix*, and what we call *Velocity*. For the *Vis Acceleratrix* bears the ſame Relation to the *Vis Motrix*, that Velocity does to Motion; and as the Quantity of Motion ariſes from the Multiplication of the Velocity into the Quantity of Matter, ſo does the *Vis Motrix* from the Multiplication of the *Vis Acceleratrix* into the Quantity of Matter. For the Sum of the Actions of the *Vis Acceleratrix* upon all the Particles of a Body, is the *Vis Motrix* of the whole; as the Sum of the *Momenta*, ariſing from the Multiplication of the Velocity into all the Particles of a Body, is the compleat *Momentum*, or Quantity of Motion of the whole: So that, as in Bodies of different Magnitudes, when the Velocity is the ſame in all of them, the Quantities of Motion are proportional to the Magnitudes or Bulks; ſo in all Bodies, when the accelerating Gravity is the ſame (that is, near the Surface of the Earth) the *Gravitas Motrix*, or the *Pondus*, is proportional to the Quantities of Matter, that is (in Homogeneal Bodies) to the Magnitudes of them. But then, aſcending higher into the Regions where the accelerating Gravity is leſs, the

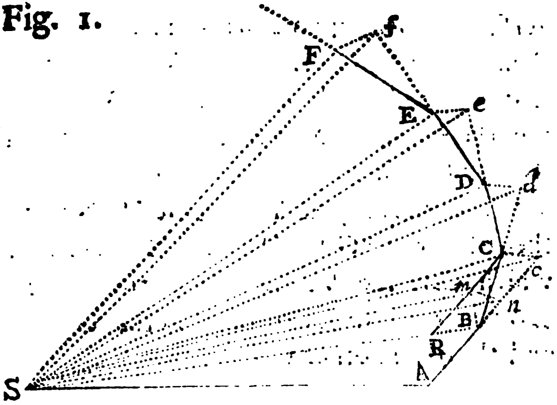
Pondus of the Body is leſſen'd in proportion to that, even as the *Momentum* or Gravity of Motion in a Body is ſaid to be leſs, in proportion to the Diminution of the Velocity with which it moves. But univerſally, as in one Caſe the *Momenta* are as the Velocities multiplied into the Quantities of Matter; ſo in the other, the *Vires* or *Gravitates Motrices*, or *Pondera* are as the accelerating Gravities into the Quantities of Matter. Thus if the accelerating Gravity in the Region *B* be twice leſs than it was in the Region *A*, then the *Pondus* of the ſame Body *M* ſhall be in the Region *B*, but half what it was in the Region A; but if the Body *N* be twice or thrice leſs than the Body *M*, then the Weight of *N* will be 4 or 6 times leſs in the Region *B*, then it was in *A*.

Suppoſe $G =$ the accelerating Gravity of the Body M in the Region A, and, the Qnantity p denoting either an Integer or a Fraction, Gp ſhall expreſs the acclerating Gravity of M in the Region B, according as it is greater or leſs there then in the Region A. And becauſe the acclerating Gravities of all Bodies in the ſame Places or at the ſame diſtances, are equal; therefore theſe Quantities G and Gp ſhall expreſs the acclerating Gravities of the Body N, Poſited at the ſame Diſtances, in the Regions A and B, with the Body M. Therefore $M \times G$ is the *Pondus* of the Body M in the Region A, and $N \times Gp$

is the *Pondus* of the Body N in the Region B, univerſally. Now Suppoſe M : N :: 2 : 1. and $p = \frac{1}{2}$; then $M \times G : N \times G p :: M \times G : \frac{1}{2} M \times \frac{1}{2} G :: 1 : \frac{1}{4}$ to that the *Pondus* of M in the Region A is quadruple the *Pondus* of N (a Body half as big) in the Region B, where the accelerating Gravity is $\frac{1}{2}$ of that in the Region A. Suppoſe M : N :: 3 : 1. then $M \times G : N \times G p :: M \times G : \frac{1}{3} M \times \frac{1}{2} G :: 1 : \frac{1}{6}$, in which Caſe the Weight of N in the Region B, is 6 times leſs than that of M in the Region A; and the like in other Caſes.

Prop:

Fig. 1.



Prop. I. *Theor.* I.

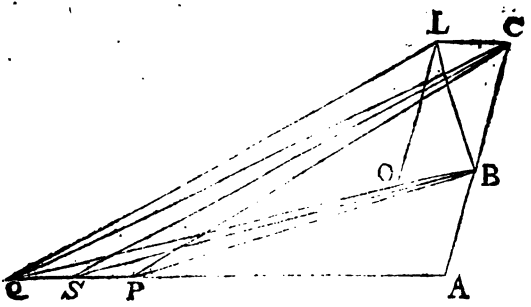
The Area's *which revolving Bodies deſcirbe (by Rays drawn to an immovable Center of Force) do lie in immovable Planes, and are proportionabce to the Times in which they are deſcrib'd.*

LET SABCDEFS be (*FIG.* I.) any Polygon, and the Lines AS, BS,*&c.* drawn to the fix'd Point S. Produce the ſides AB,BC,CD,DE,EF to the the Point *c*, *d*, *e*, *f*, in ſuch a Manner that AB may=B*c*, BC=C*d*, CD=D*e*, DE=E*f*. From the Point C draw CR to cut BS in R, and let it be parallel to *c* B, and C*c* parallel to BS; ſo likewiſe draw D*d* parallel to SC, and E*e* to SD, and F*f* to SE. The Triangles SBC, SB*c* are equal being upon the ſame Baſe SB, and between the ſame Parallel's SB and C*c*. Alſo ſince AB=B*c*, the Triangles ASB, BS *c* are equal, therefore ASB=BSC. So in like manner 'twill be prov'd, that the Triangle CSD is =CS*d* being upon the ſame Baſe CS, and between the ſame Parallel's CS, D*d*; but becauſe BC=C*d*, the Triangle BSC=CS *d*, therefore BSC=CSD, and conſequently ASB=CSD; and by the ſame way of arguing we ſhall conclude all the Triangles of the Polygon to be equal to one another, *viz.* ASB=BSC=CSD=DSE=ESF, *&c.* Now I ſhall demonſtrate that in equal Times, a Body (urg'd by a centripetal Force tending to the Center S) will deſcribe theſe equal *Area's*. For let T expreſs any Portion of Time divided into ſeveral equal Parts expreſs'd by A. B. C. D. E. *&c.* and let us imagine a Body plac'd in the Line AB at the Point A, and to be carried by ſuch a Force that in the firſt part of time A it wou'd deſcribe the Line AB. It's plain (by the firſt Law of Motion) that if the Body were not otherwiſe hinder'd in the 2d part of Time B, it wou'd deſcribe the Line B*c*=AB, and ſo the *Area's* ASB, BS *c* wou'd be equal. But when it is come to the Point B imagine the centripetal Force to exert it ſelf and begin to cauſe the Body to tend towards the Center S (which we will ſuppoſe to be the Center of Forces) and let this centripetal Force be ſuch, that if the Body were entirely acted by that, it wou'd make it deſcribe the Line RB in the Time B. Here then the Body at the Point B is drawn by two Forces, the one its innate Force whoſe Direction is from B to *c*, the other the Centripetal, whoſe Direction is from B to R. Therefore (by *Corol.* I. of the Laws of Motion) the Body will move in the Line BC the Diagonal of the Parallelogram RBC*c*; and at the end of the Time B it will be found in the Point C, ever in the ſame Plane with the Triangle ASB. In like manner in the 3d part of Time C the Body will deſcribe the Line CD the Diagonal of the Parallelogram of which the Triangle CD*d* is the ½: And in the 4th part of Time 'twill deſcribe the Diagonal DE, and ſo of the reſt: So that by the Conjunction of theſe two Forces it will deſcribe the Periphery of the Poligone ABCDEF. Now the Triangles ASB, BSC, CSD, *&c.* were proved to be equal; but they are deſcrib'd in the equal times A, B, C, *&c.* therefore equal *Area's* are deſcribe in equal Times; and conſequently Time A : Time $\overline{A+B}$ :: *Area* ASB :: *Area* $\overline{ASB+BSC}$, and Time A : Time $\overline{A+B+C}$ :: *Area* ASB : *Area* $\overline{ASB+BSC+BSD}$, and ſo on; that, is the *Area* deſcrib'd are as the Times of Deſcription. Now if the Lines AB, BC, CD, *&c.* the Caſes of the Triangles be diminiſh'd, and their Number augmented infinitely, the Parameter of the Poligone will become a Curve Line, and ſo the Centripetal Force (by which the Body is drawn out of the Tangents) will act inceſſantly, and without Intermiſſion. And any *Area's* deſcrib'd (which in the former Caſe were ſhewn to be proportional to the Times of Deſcription) will ſtill continue proportional to the ſame Time in this Caſe alſo. Q: E: D.

COR. I.

In *Mediums* where a Body meets with no Reſiſtance, if the *Area's* are not proportional to the Times, the Forces do not tend to the common Concourſe of the Rays. I ſay, in non-reſiſting *Mediums*: For in *Mediums* where a Body meets with Reſiſtance, and ſo is any way hinder'd in its Motion, the *Area's* may poſſibly not be proportional to the Times, and yet the Forces may tend to the common Concourſe of the Rays. *Ex. gr.* Suppoſe in (*FIG.* I.) the Body to have deſcrib'd the Line AB in the Time t, and whereas (if not otherwiſe hinder'd) it wou'd deſcribe $Bc=AB$ in the next equal Portion of Time, imagine it to meet with ſome Reſiſtance, ſo as to deſcribe no more than Bn in the ſame Time; then the Triangle $SBn=SBm$ will be leſs than SBA; ſo that the centripetal Force acting as before in the Caſe where we ſuppos'd no Reſiſtance, the *Area's* will not now be proportional to the Times, and yet the Forces tend to the Point S the common Interſection of the Rays.

Fig. 2.



Cor. II.

In all *Mediums* whatſoever, if the Deſcription of the *Area's* be accelerated (that is, if an equal *Area* be deſcrib'd in a leſs Portion of Time, or a greater *Area* in an equal Time) then the Forces do not tend to the common Concourſe of the Rays, but decline from thence in *Conſequentia* (that is, to the Parts towards which the Body is moving.

For (*FIG.* II.) AB being =B *c*, the *Area's* ASB, BS *c*, taken about the ſame Point S, will ever be equal. But if we took a Point as P between S and A, the *Area's* ASB wou'd be greater than the *Area* BP *c*. On the other Hand, if we take a Point as Q on the other Side of S, the *Area* BQ *c* will be greater than ASB. Therefore if when in the firſt Portion of Time the *Area* ASB was deſcrib'd, in the 2d Time a greater *Area*, as BQL (=BQ*c*) be deſcrib'd; 'tis certain, that the centripetal Force muſt have acted with a Direction, as LC parallel to B*o*Q, that is, that the Forces now tend to the Point Q. The ſame may be ſhewn for any other Point taken in the ſame Right Line beyond Q; if the Direction of the *Area's* be accelerated, the Tendency of the Forces will ſtill be in *Conſequentia*.

THEOR. II.

Every Body moving in a Curve, and (by Rays drawn to a certain Point, either immovable, or going on uniformly in a Right Line Motion) deſcribing Area's *about that Point proportional to the Times, is urged by a Centripetal Force tending to that ſame Point.* (See *FIG.* I.)

CASE I.

FOR every Body moving in a Curve, is carried out of its Right Line Courſe by ſome Force acting upon it, by the firſt *Law of Motion*. And that Force by which the Body is made to deflect from its Rectiline Courſe, and to deſcribe the little equal Triangles SAB, SBC, SCD, in equal Times, about the immovable Point S, muſt act in the Point B with a Di-

rection

rection parallel to the Line C*c*, by the Second *Law of Motion:* That is, it acts with the Direction BS. So in the Point C it acts according to a Line parallel to D*d*, that is, by the Line CS, for the ſame Reaſon: And ſo 'tis every where. Therefore it always acts according to Lines that tend to the immovable Point S. Q. E. D.

CASE II.

If the Plane in which the Body deſcribes a Curvilineal Figure (and conſequently the Center or Point to which the Forces tend) moves uniformly in a Right-line Direction; 'tis all one as to the Effect (by *Coroll.* 4. to the general *Laws of Motion*) whether it does ſo, carrying the Body and All along with it, or whether it be abſolutely at reſt. So that either way the Truth of the Propoſition is manifeſt. Q. E. D.

SCHOL.

'Tis poſſible that a Body may be urg'd by a Centripetal Force, compounded of ſeveral other Forces; about which Compoſition of Forces we have ſpoke ſufficiently already in the firſt Pages of this Book. Now always in ſuch Caſes as this, the Senſe of the foregoing *Theorem*, is, that the Force which is compounded of all the reſt, is that which tends to the Center. *Ex. gr.* Suppoſe in *FIG.* II. *Pag.* 7. where the Force acting with the Direction CB is ſhewn to be equivalent to the Forces acting with the Directions CD, CE, CF. We are to conceive the compound Force, whoſe Direction is CB to tend to the Center; ſo that the Center of the Forces, where all the Rays meet, is ſomewhere or other in the Line CB (produced at leaſt.

'Tis further to be noted, that if there be any Force that acts with a Direction perpendicular to the Superficies deſcrib'd by the Body, that this Force may be altogether left out of the Account of the Compoſition of the Forces. For tho' it makes the Body to decline from the Plane of its Motion, yet it tends no way to alter the Magnitude of the Surface deſcrib'd, neither as to the encreaſing or leſſening of it, and may therefore be neglected.

THEOR. III.

Every Body (which by a Ray drawn to the Center of another Body (howſoever mov'd) deſcribes Area's *about that Center proportional to the Times) is urged with a Force compounded of the Centripetal Force tending to that other Body, and of all the accelerating Force with which that other Body it ſelf is urg'd.*

LET the Body M deſcribe *Area's* proportional to the Times, about the Center of the Body N; and let the Body N be urged with an accelerating Force, as +A. Suppoſe now both Bodies to be drawn in a parallel Direction by a new Force equal and contrary to the former, which put =—A. Then, notwithſtanding, the Body M will deſcribe *Area's* proportional to the Times, as it did before the Action of this new Force, with reſpect to N, (by *Cor.* 5. to the *Laws of Motion*;) but the Force —A will deſtroy the Force +A in the Body N (which will now therefore either quieſce, or move uniformly in a Right Line (by the firſt *Law of Motion.*) But alſo becauſe the Body M is influenced by this new Force —A; therefore the Force by which it at firſt tended to the Body N being call'd C, the Force by which it is now urged ſhall = C—A; which Force (by *Theor.* II.) ſhall ſtill tend to the Center of the Body N, becauſe we have ſhewn, that the *Area's*, with reſpect to that Center, are ſtill proportional to the Times. So that every Body, *&c.* Q: E: D.

COR. I.

Hence if one Body, with a Ray drawn to a[illegible]her, deſcribes *Area's* proportional to the Times; and if from the whole Force (whether a ſimple or compounded one) with which the former Body is urged, we ſubſtract the whole accelerating Force with which the latter Body is urged; all the remaining Force by which the former Body is urged will tend to the latter Body, as its Center. This is plain from that Demonſtration; for the reſulting Force is = C—A.

C o r. II.

If the *Area's* be not accurately, but only nearly proportional to the Times, then the remaining Force will not accurately, but nearly tend to the other Body. And the Converſe of this is true alſo.

C o r. III.

If a Body with a Ray drawn to another Body deſcribes *Area's*, which compar'd with the Times, are very unequal; and that other Body is either quieſcent, or moves uniformly in a Right Line: Then, either there is no Action of the Centripetal Force tending to that other Body as its Center, or elſe its mix'd and compounded with ſome very ſtrong Actions of other Forces: And the whole Force(compounded of all, if there are many) is directed to another Center, either a movable or an immovable one, about which the Deſcription of the *Area's* is equable and regular. And the ſame holds, when that other Body is moved even with any ſort of Motion; provided the Centripetal Force (that which we here ſay is directed to another Center) be taken for the Remainder, after the Subſtraction (of the whole accelerating Force that acts upon the latter Body) from the real Centripetal Force of the former Body.

The Reaſon of all which is eaſily deducible from the Second *Theorem* (foregoing) in Conjunction with this Third. For where the *Area's* deſcrib'd are proportional to the Times, there the Action of the Centripetal Force tends to a Center, that is, either quieſcent or moves uniformly in a Right Line: This is plain from *Theor.* II. Therefore if that Center be either quieſcent, or moves uniformly in a Right Line; and the *Area's* deſcrib'd be not proportional to the Times, 'tis a ſure Proof that either the Action of the Centripetal Force does not tend to that Center at all, or elſe that 'tis confounded and diſturb'd by the powerful Actions of other Forces. And if according to the Tenour of *Theor.* III. the Center here ſpoken of be another Body, the Reaſon of the preſent *Corollary* will be ſufficiently clear and manifeſt.

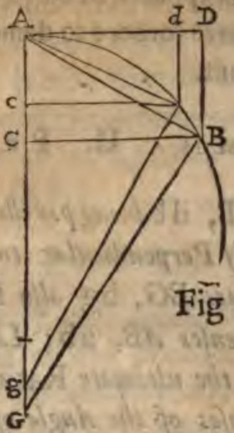


Fig 3.

LEMMA I.

Let AB be any Arch of a Curve, (FIG. III.) *the Right Line AB its Subtenſe, and AD its Tangent equal and parallel to the Ordinate* CB. *When the Points A and D come infinitely near together, the Arch AB, the Subtenſe AB and the Tangent AD will be equal to one another; or in the* Newtonean *Style, the laſt* Ratio *of the Arch, the Chord and Tangent will be a* Ratio *of Equality. Each one of theſe Lines therefore may be uſed or taken for the other in all Argumentatious about ultimate* Ratio's; *that is, when theſe Quantities are conſider'd as vaniſhing upon the infinitely near Approach of the Points A and B together.*

This *Lemma* appears to be ſo clear and evident, that I have forbore to demonſtrate it upon that Account.

LEMMA II. PROB.

The Lines DB, db *being parallel to AC, and conſequently Perpendicular to AD; drawing the Lines BG,* bg *alſo Perpendicular to the Subtenſes AB,* ab; *Let it be required to find the ultimate* Ratio *of DB,* db *(the Subtenſes of the Angle of Contact) to the conterminal Arches AB,* ab; *becauſe of the Right Angles* Abg, *ABG, a Circle paſs thro' the Points* Abg, *as alſo another Circle thro' the Points* A,B,G,

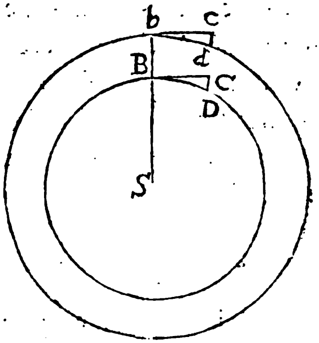
LET the Arch $AB = A$, $Ab = a$ $AG = 2R$; $Ag = 2r$. $AC = X$; $Ac = x$. $BC = Y$. $bc = y$, $AB = D$ and $Ab = d$. Now becauſe of the Circles we have theſe Equations $YY = 2RX - XX$, and $DD = 2RX$; $yy = 2rx - xx$ and $dd = 2rx$; and dividing by $2R$ and $2r$ (which are each of them a ſtanding Quantity) DD will always be as X, and dd as x. But when the Points A,B, and A,*b* approach infinitely near, the Quantity X becomes $\dot{X}$, and x becomes $\dot{x}$, and are the Subtenſes of their reſpective Angles of Contact, alſo the Chord D is in this Caſe coincident with the Arch A, and the Chord *d* coincident with the Arch *a*; therefore $2R\dot{x}$ is $= A^2$ and A^2 is as $\dot{x}$, likewiſe $2r\dot{x} = aa$, and aa is as $\dot{x}$, ſo that the Subtenſes of the Angles of Contact are as the Squares of their conterminal Arches. Q. E. I.

SCHOL.

We have only here conſider'd *db* and DB (the Subtenſes of the evaneſcent Angles of Contact) as Perpendiculars to the Tangent AD; but it will be the ſame thing if we ſuppoſe them to be inclined, for let AD and BD make any given Angle, then the ultimate *Ratio* of BD to *bd* will be the ſame as before, and conſequently the ſame with that of AB*q* to *abq*; If the Angle D be not given, yet the Angles D,*d* will always converge to, and come nearer an Equality, than by any Difference aſſignable, and conſequently will at laſt be equal; therefore the Lines BD, *bd* are in the ſame *Ratio* to one another as before.

THEOR.

Fig. 4.



THEOR. IV.

Bodies deſcribing different Circles with an equable Motion; their Centripetal Forces tend to the Centers of thoſe Circles; and to the Law of thoſe Forces may alſo be diſcover'd. (FIG. IV.)

1. THAT the Centripetal Forces tend to the Centers of theſe Circles, is moſt evident. For ſince the Motion in the Peripheries is ſuppoſed equable, therefore from the common Principles of Mechanicks, the Arches run will be as the Times in which they are run. But the Sectors (or *Area's* taken from the Centers of the Circles) are in Proportion as the Arches upon which they ſtand. Therefore thoſe *Area's* are as the Times; and therefore (by *Theor.* II.) the Centripetal Forces tend to the Centers.

2. Imagine the Bodies to deſcribe the Arches BD, *bd* in the ſame time, and let CD, *cd* be drawn parallel to SB*b*, that is, perpendicnlar to the reſpective Tangents BC, *bc*. Now ſince 'tis by the Centripetal Forces that the Bodies are drawn out of the Tangents into the Circumferences, 'tis plain that thoſe *Lineolæ* CD, *cd* (conſider'd not as of any finite Length) will repreſent the Centripetal Forces: For they are the genuine Effects of thoſe Forces, produced in the ſame time, and therefore are proportional to them. Again, becauſe they are *Lineolæ Naſcentes*, they will not differ from others that tend to the Centers of the Circles, whither the Centripetal Forces ought to be directed, by the former Part of this *Prop.* III.

3. Now to diſcover the Law of theſe Forces, let S=R. SB=*r*. *bd*=A, BD=*a*. *cd*=$\dot{\mathrm{X}}$ (Subtenſe of the Angle of Contact, in the greater Circle) CD=$\dot{x}$ (Subtenſe of the Angle of Contact) in the leſs. By *Lem.* II. we have $2\mathrm{R}\dot{\mathrm{X}}=\mathrm{AA}$. and $2r\dot{x}=aa$, ſo that $\mathrm{A}^2 : a^2 :: 2\mathrm{R}\dot{\mathrm{X}} : 2r\dot{x}$, conſeqnently $\dot{\mathrm{X}} : \dot{x} :: \frac{\mathrm{A}^2}{\mathrm{R}} : \frac{a^2}{r}$. That is this THEOR. viz. The Centripetal Forces are as the Squares of the Arches (deſcrib'd in the ſame Time) divided by the reſpective *Radii*. Q: E: I. In order to the Inveſtigation of the many noble *Corollaries* that flow from this *Theorem*, we are obliged to introduce theſe Symbols. C= Centripetal

Force. T=Time. V = Velocity in the outer Circle *bd*; and *c*, *t*, *v*, to expreſs the like Quantities in the inner Circle BD. By the *Theorem* 'tis $C : c :: \frac{A^2}{R} : \frac{a^2}{r}$; but becauſe the Arches A, *a*, are deſcrib'd in the ſame Time, therefore, by the Principles of *Mechanicks*, the Velocities are as the Spaces, that is, the Arches run; therefore $V : v :: A : a$, therefore $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$, that is COR. I. The Centripetal Forces are as the Squares of the Velocities, divided by the *Radii*. This is the Law for the Centripetal Forces, in the Caſe of infinitely ſmall Arches deſcrib'd in the ſame Time. Let T, *t*, expreſs now the whole Periodick Times of deſcribing the Circles, and let the Periphery of the outer Circle =P, and that of the inner =*p*, and all the reſt as before. From *Mechanicks*, the Times of deſcribing the Peripheries *P*, *p*, are in the *Ratio* compounded of the direct *Ratio* of thoſe Peripheries (or the Spaces run) and the Reciprocal of the Velocities, that is, $T : t :: P \times v : p \times V$; but $P : p :: R : r$ (the Peripheries are as the *Radii*) therefore $T : t :: R \times v : r \times V$, but alſo $V : v :: A : a$ (the Velocities in the equable circular Motions are eternally as the Arches deſcrib'd in the ſame Time) therefore $T : t :: R \times a : r \times A$, and $T r A = t R a$, and (ſquaring) $T^2 r^2 A^2 = t^2 R^2 a^2$, therefore

$\frac{A^2}{R} : \frac{a^2}{r} :: \frac{t^2}{r} : \frac{T^2}{R}$, but (by the *Theor.*) $C : c :: \frac{A^2}{R} : \frac{a^2}{r}$, therefore $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$, or, which is all one, $C : c :: R \times t^2 : r \times T^2$, that is COR. II. The Centripetal Forces are in the *Ratio* compounded of the direct *Ratio* of the *Radii*, and the reciprocal *Ratio* of the Squares of the Periodick Times. Which is the Law for the Centripetal Forces, when we conſider the whole Periodick Times. Now, from theſe general Laws all the others, relating either to Centripetal Forces or Velocities, are eaſily deduced, making any Suppoſition whatſoever. For, whatever the Periodick Times be, we have got a general *Theorem* for the Centripetal Forces in this laſt *Corollary:* And as for the Velocities, they are ever as the Arches A, *a*, deſcrib'd in the ſame Time, and conſequently will be found by the Proportion of thoſe Arches.

Ex. gr. 1. Let us ſuppoſe $T = t$. Now ſince (by *Cor.* II.) 'tis $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$, and $t = T$ (by Hypotheſis) therefore $C : c :: \frac{1}{r} : \frac{1}{R} :: R : r$. That is THEOR. If the Periodick Times are equal, the Centripetal Forces are directly as the Rays. For the Velocities. 'Tis $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$

(by *Cor.* I.) and C : *c* :: R : *r* (in this Caſe of equal Times.) Therefore $\frac{V^2}{R} : \frac{v^2}{r} :: R : r$. from whence V : *v* :: R : *r*. *Theor.* The Velocities are directly as the Rays, when the Periodick Times are equal.

2. Let $T^2 : t^2 :: R : r$. Therefore ſince $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$ (by *Cor.* II.) 'tis plain, that C=*c*. *Theor.* If the Squares of the Periodick Times be directly as the Rays, the Centripetal Forces are equal. For the Velocities. 'Tis $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$ (by *Cor.* I.) and C=*c* (in this Caſe;) thererefore $\frac{V^2}{R} = \frac{v^2}{r}$, from whence $V^2 : v^2 :: R : r$, and $V : v :: R^{\frac{1}{2}} : r^{\frac{1}{2}}$. *Theor.* The Velocities are directly in the ſubduplicate *Ratio* of the Rays, when the Squares of the Periodick Times are as the Rays.

3. Let $T^2 : t^2 :: R^2 : r^2$. Therefore ſince $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$ (by *Cor.* II.) 'Tis clear, that C : *c* :: *r* : R. *Theor.* If the Squares of the Periodick Times be as the Squares of the Rays directly, the Centripetal Forces ſhall be as the Rays reciprocally. For the Velocities. 'Tis $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$ (by *Cor.* I.) and C : *c* :: *r* : R (in this

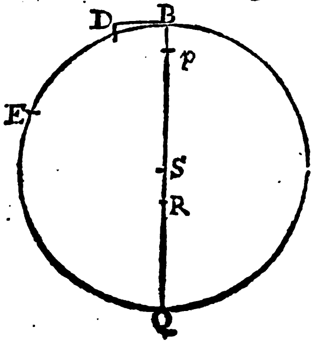
Caſe) therefore $\frac{V^2}{R} : \frac{v^2}{r} :: r : R$, from whence $V^2 = v^2$, and $V = v$. Theor. If the Squares of the Periodick Times are as the Squares of the Rays, the Velocities are equal.

4. Let $T^2 : t^2 :: R^3 : r^3$. Therefore ſince $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$ (by *Cor.* II.) 'tis plain, that $C : c :: r^2 : R^2$. Theor. If the Squares of the Periodick Times be as the Cubes of the Rays, the Centripetal Forces ſhall be reciprocally as the Squares of the Rays. For the Velocities. 'Tis $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$ (by *Cor.* I.) and $C : c :: r^2 : R^2$ (in this preſent Caſe) therefore $\frac{V^2}{R} : \frac{v^2}{r} :: r^2 : R^2$, from whence we have $V^2 R = v^2 r$, and $V : v :: r^{\frac{1}{2}} : R^{\frac{1}{2}}$. Theor. When the Squares of the Periodick Times are as the Cubes of the Rays, the Velocities are reciprocally in the Subduplicate *Ratio* of the Rays.

5. Let $T^m : t^m :: R^n : r^n$ univerſally. Therefore ſince $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$ (by *Cor.* II.) and $T^2 : t^2 :: R^{\frac{2n}{m}} : r^{\frac{2n}{m}}$ (by the Conſequence of this Suppoſition) therefore $C : c :: \frac{r^{\frac{2n}{m}}}{r} : \frac{R^{\frac{2n}{m}}}{R}$, that

is, $C : c :: r^{\frac{2n-m}{m}} : R^{\frac{2n-m}{m}}$. For the Velocities. We have $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$ (from *Cor.* I.) and $C : c :: r^{\frac{2n-m}{m}} : R^{\frac{2n-m}{m}}$, therefore $\frac{V^2}{R} : \frac{v^2}{r} :: r^{\frac{2n-m}{m}} : R^{\frac{2n-m}{m}}$; from whence (by juſt Operation with the Exponents) we ſhall have $V : v :: r^{\frac{n-m}{m}} : R^{\frac{n-m}{m}}$. So that if the Exponent of the Periodick Times be m, and that of the Rays be $\pm n$; then the Exponent of the Centripetal Forces ſhall $= \frac{2n-m}{m}$, and that of the Velocities ſhall $= \frac{n-m}{m}$. Q. E. I.

Fig. 5.



PROB.

PROB. I.

To determine the Space the Centripetal Force wou'd carry a Body thro', in the ſame Time that it deſcribes any given Arch of a Circle, in which 'tis ſuppoſ'd to move uniformly.

IMagine the Body at the Point B (*FIG.* V.) to be acted entirely by the Centripetal Force, and ſo drawn on by it in the Diameter BQ towards the Center S. Now, if this Force be ſuppos'd to remain ſtill the ſame, without any manner of Alteration, and to act upon the Body by continual repeated Impulſes; then the Body will be carry'd by it after the ſame manner, and according to the ſame Law, that it wou'd be by the Force of Gravity. And therefore the Body (thus acted upon by the Centripetal Force) wou'd deſcribe Spaces in the Duplicate *Ratio* of the Times: For that is the *Law of Gravity* (by *Theor.* II. *Galil. Dial.* III.)

This premis'd; Let the infinitely ſmall Arch BD = a, and the Particle of Time 'tis deſcrib'd in be put = t; alſo let the given Arch (intended in the *Problem*, be BE) = and the Time 'tis deſcrib'd in be = T. becauſe the Motion in the Circle equable, 'tis $t : T :: a : A$, and t

A^2, (from what *Galilæus* has demonſtrated about equable Motions.) Alſo (putting the Diameter of the Circle = D) 'tis $a^2 : A^2 :: \frac{a^2}{D} : \frac{A^2}{D}$; therefore $t^2 : T^2 :: \frac{a^2}{D} : \frac{A^2}{D}$. But it was ſhewn before, that the Spaces which the Body (acted by the Centripetal Force) deſcribes, are as the Squares of the Times; and therefore thoſe Spaces are as the Quantities $\frac{a^2}{D}$, and $\frac{A^2}{D}$. But in the Time t, while the little Arch a is deſcrib'd, the Centripetal Force carries the Body through the little Space $\frac{a^2}{D}$ (by *Art.* II. *Theor.* IV.) therefore in the Time T, while the Arch A is deſcrib'd, it wou'd (if the Body were ſolely acted by it) carry it thro' the Space $\frac{A^2}{D}$. From whence ariſes this THEOR. That in what Time a Body revolving in a Circle deſcribes any given Arch BE, in the ſame Time, if it were carried by the Centripetal Force in a Rectilineal Courſe, it wou'd deſcribe a Space equal to $\frac{BEq}{BQ}$, that is, to the Square of that Arch divided by the Diameter. Q. E. I.

Cor. I.

Bodies deſcribing different Circles with equable Motions; the Spaces (they wou'd be carried thro' by the Centripetal Forces in thoſe Circles, in the ſame Time) are directly proportional to the Centripetal Forces themſelves. *Ex. gr.* Let D. *d.* be the Diameters of Two Circles, having their Center common; *a. b.* Two infinitely ſmall Arches deſcrib'd in theſe Circles in the ſame Particle of Time$=r$. A. B. any Two Arches of a finite Length deſcrib'd, the one in one Circle, and the other in the other, in the ſame Time T. Then from what was ſhewn before $t^2 : T^2 :: \frac{a^2}{D} : \frac{A^2}{D}$, for one Circle; and alſo $t^2 : T^2 :: \frac{b^2}{d} : \frac{B^2}{d}$, for the other Circle. Therefore (by Equality) $\frac{a^2}{D} : \frac{A^2}{D} :: \frac{b^2}{d} : \frac{B^2}{d}$, and $\frac{a^2}{D} : \frac{b^2}{d} :: \frac{A^2}{D} : \frac{B^2}{d}$. But the little Arks *a* and *b* being deſcrib'd in the ſame Time (by the Hypotheſis) the Quantities $\frac{a^2}{D}$, $\frac{b^2}{d}$ expreſs the Proportion of the Centripetal Forces in thoſe Circle (by *Art* III. *Theor.* IV.) Alſo the Quantities $\frac{A^2}{D}$, $\frac{B^2}{d}$ are the Spaces the Bodies are carry'd thro' by the Centripetal Forces in thoſe Circles in the Time T (by the Suppoſition.) Therefore the Conſequence is clear.

Cor. II.

Suppoſing all as before; the Spaces that the Bodies wou'd be carry'd thro' by the Centripetal Forces, in unequal Times, are in the *Ratio* compounded of the Squares of the Times, and the Centripetal Forces. All the other Symbols ſtanding, let the Arch B be deſcrib'd in the Time θ; the infinitely ſmall Arks a and b being ſuppoſed to be deſcrib'd in the ſame Particle of Time t, as before. Then $t^2 : T^2 :: \frac{a^2}{D} : \frac{A^2}{D}$, for one Circle, and $t^2 . \theta^2 :: \frac{b^2}{d} : \frac{B^2}{d}$, for the other Circle. Therefore (equalling theſe Two Values of t^2, and making the Analogies from thence) we have $\frac{A^2}{D} : \frac{B^2}{d} :: T^2 \times \frac{a^2}{D} : \theta^2 \times \frac{b^2}{d}$. But the Two former Terms of the Analogy are the Spaces the Bodies wou'd be carry'd thro' by the Centripetal Forces in the Time T and θ; and the Two latter Terms are compounded of the Squares of thoſe Times, and the Centripetal Forces in the Two Circles. Therefore, *&c.*

C o r. II.

If the Symbols A and B, inſtead of Arches, denote the whole Circumferences of the Circles, and conſequently T and θ be the whole Periodick Times of the Deſcription of thoſe Circumferences; then 'tis evident, that the Quantities $\frac{A^2}{D}$, $\frac{B^2}{d}$, will be to one another as $D : d$, that is, the Spaces (the Centripetal Forces wou'd carry the Bodies thro' in the Times that the Circumferences are deſcrib'd) wou'd be in Proportion as the Diameters, or the Circumferences. For $A : B :: D : d$ (that is, the Peripheries are as the Diameters) and ſo $\frac{A^2}{D} : \frac{B^2}{d} :: \frac{D^2}{D} : \frac{d^2}{d} :: D : d$. And there's no need of any Proof that this muſt be true univerſally, whatever be the *Ratio* of the Periodick Times, or in what unequal Times ſoever the Circumferences of the Circles are imagined to be deſcrib'd by the Two Bodies. However, if there were, the *young Student* for his Exerciſe might eaſily clear it, from the general Analogy in the laſt *Corollary*. For there 'twas ſhewn, that $\frac{A^2}{D} : \frac{B^2}{d} :: T^2 \times \frac{a^2}{D} : \theta^2 \times \frac{b_2}{d}$: Now, the Quantities $\frac{a^2}{D}$, $\frac{b^2}{d}$ expreſſing the Centripetal Forces in theſe Circles, let us uſe the Symbols C. *c.* for them, and then the Analogy ſtands thus, *viz.* $\frac{A^2}{D} : \frac{B^2}{d} :: T^2 \times C : \theta^2 \times c$. Now, here let the Periodick Times be in what *Ratio* of the Rays we pleaſe, we ſhall ſtill find $\frac{A^2}{D} : \frac{B^2}{d} :: D : d$. *Ex. gr.* Let the Periodick Times be equal, that is, $T = \theta$, then will $C : c :: D : d$ (by what was ſhewn above at the *Corollaries* of *Theor.* IV.) and conſequently $\frac{A^2}{D} : \frac{B^2}{d} :: D : d$. Again, let $T^2 : \theta^2 :: D : d$, then will $C = c$, and ſo $\frac{A^2}{D} : \frac{B^2}{d} :: D : d$. Let $T^2 : \theta^2 :: D^3 : d^3$, then will $C : c :: d^2 : D^2$, and conſequently $\frac{A^2}{D} : \frac{B^2}{d} :: D^3 \times d^2 : d^3 \times D^2 :: D : d$. And ſo proportionably in any other Suppoſition that may be made.

COR. IV.

From what is found in this *Problem*, we may eaſily compare Centripetal Forces with any other known Force, ſuch as that of Gravity (*Ex. gr.*) which ſhall be the Buſineſs of the following *Problem*.

PROB. II.

To find the Proportion of the Centripetal Force to that of Gravity.

USing the ſame Symbols as in *Prob.* I. ſuppoſe in the Particle of Time t, while the little Ark BD $= a$ is deſcrib'd, (*FIG.* V.) the Body by the Force of its Gravity deſcrib'd the little Space BP, which put $= p$. Then in the Time T, while the Arch BE $=$ A is deſcrib'd, the Body will have deſcended through a Space $= \frac{T^2 \times p}{t^2}$ (for $t^2 : T^2 :: p :$ to the Space deſcrib'd in the Time T, by the *Law of Gravity*) $=$ BR (*Ex.gr.*) But $\frac{T^2 \times p}{t^2} = \frac{A^2 \times p}{a^2}$ (ſince $t^2 : T^2 :: a^2 : A^2$, becauſe of the equable Motion in the Circle) So that BR $= \frac{A^2 \times p}{a^2} =$ the Space deſcrib'd by the Gravity in the Time T. But in the ſame Time, the Centripetal Force wou'd carry the Body thro' a Space $= \frac{A^2}{D}$ (by what was ſhewn at *Prob.* I.) Therefore theſe Spaces are to one another as $\frac{A^2 \times p}{a^2} : \frac{A^2}{D}$, that is, as $\frac{p}{a^2} : \frac{1}{D}$, or as $D \times p : a^2$. And conſequently the Force of Gravity, and the Centripetal Force, making the

Body to deſcribe theſe Spaces in the ſame Time, are one to another in the ſame *Ratio*. Q. E. I.

But becauſe the Terms p and a are infinitely ſmall, therefore to expreſs this Proportion in finite Quantities, we may proceed thus. Let it be $t^2 : T^2 :: p : k$; then k is the Space deſcrib'd by the deſcending Body in the Time T. But (from the Uniform Motion in the Circle) $t^2 : T^2 :: a^2 : A^2$, therefore (by Equality) $a^2 : A^2 :: p : k$, and $a^2 : p :: A^2 : k$, and conſequently inſtead of the Terms p and a^2, putting in k and A^2, which are proportional to them, the Proportion will be as $D \times k : A^2$. So that we have this *Theorem*, The Force of Gravity is to the Centripetal Force (of a Body revolving uniformly in a Circle) as the Rectangle under the Diameter of the Circle, and the Line deſcrib'd by a deſcending Body in any given Time, to the Square of the Arch deſcrib'd by the revolving Body in the ſame Time. Q. E. F.

COR.

When $D \times k = A^2$, then thoſe Two Forces are equal; that is, when $k = \frac{A^2}{D}$, or the Space a Body deſcribes by the Force of Gravity, is equal to that it would be carried thro' by the Centripetal Force in the ſame Time.

THEOR. V.

A Body revolving in the Circumference of a Circle, the Centripetal and Centrifugal Forces are equal to one another.

FOR while the little Arch (*FIG.* V.) BD (*Ex. gr.*) is deſcrib'd, the Centripetal Force has carried the Body towards the Center by the Space CD. And if the Centripetal Force were away, the Body, inſtead of deſcribing the Arch BD, wou'd go on in the Tangent, and deſcribe in the ſame Time the *Lineola* BC (which by *Lemma* I. is = the little Arch BD.) In which Caſe it wou'd be carried off from the Curve of the Circle, by the Diſtance of the ſame *Lineola* CD; and this is the Effect of the Centrifugal Force. Therefore the Centripetal and Centrifugal Forces producing an Effect of the ſame Magnitude in the ſame Time, are equal to one another. Q. E. D.

COR. I.

All the *Theorems* relating to Centripetal Forces, which were deduced in the *Corollaries* of *Theor.* IV. are applicable to Centrifugal Forces alſo.

COR.

COR. II.

And, according to the Tenour of the laſt *Problem*, the Centrifugal Force ſhall be to that of Gravity, as $A^2 : D\times k$; and conſequently equal to it, when $A^2=D\times k$. But to explain here a little what I intend by the Centrifugal Force being equal to that of Gravity, let us conceive a Body revolving in a Circle to be held by a Thread, produced out of the Center of the Circle it deſcribes, ſo that by the Means of this Thread it keeps always exactly in the Circumference of the Circle. 'Tis certain, that as the Body revolves with a greater or leſs Degree of Velocity, ſo the Thread that holds it will be ſtretch'd more or leſs violently. For the Endeavour to recede from the Center of its Motion will be greater or leſs, as the Velocity with which it moves is either greater or leſs; and that greater or leſs Endeavour of receding will produce a greater or leſs Tenſion of the Thread. This ſtretching of the Thread by the Body, as it moves in this manner circularly, is the Effect of the Centrifugal Force, as that Tenſion of it by the meer Weight of the Body, where it is freely ſuſpended by the ſame, without the Intervention of any ſort of Motion or Impediment, is the compleat Effect of the Force of

Gravity. And as the Degrees of Velocity (with which a Body may move circularly) are infinite, and conſequently there may be as many Degrees of Tenſion of the Thread that detains it in its Path in that Motion; ſo when ſuch a Degree of Velocity is pitch'd upon, that the Tenſion of the Thread ariſing from thence, is equal to that which is the Reſult of the free Suſpenſion; we ſay then, that the Centrifugal Force is equal to that of Gravity, becauſe they both produce the ſame Effects upon the Thread; or whatever Proportion the Tenſions bear to one another, we may ſay, the Forces (the Cauſes of them) bear the ſame. Now, to enquire a little into this Matter. The revolving Body moving uniformly in the Circle, we will conſider the Velocity of that Motion, as ſome one or other of thoſe Velocities, which would be acquir'd by it in a free perpendicular Deſcent, *Ex. gr.* in the Diameter of the Circle. Now, whatever Space the Body deſcribes by the Force of Gravity in its Deſcent, if it afterwards moves equably with the greateſt Velocity gotten by that Deſcent, it's a known *Theorem*, That the Space deſcrib'd by the uniform Motion ſhall be double that deſcrib'd by the uniformly Accelerate, in the ſame Time. Let the Body therefore in the Time T deſcend thro' the Space k, and if with the Velocity gotten by this Deſcent it afterwards moves equably in the Circle, and does in the ſame Time T deſcribe the Arch A; then ſhall $A=2k$. Now, let $k=mD$ univerſally, where m denotes either an Integer, or a Fraction. Then is $A=2mD$, and $\frac{A}{D}$ (the Space the Centrifugal Force would carry the Body thro' in the Time T)$=4mmD$. Therefore the Space deſcribed by the Centrifugal Force, is to that deſcribed by the Gravity in the ſame Time T, as $4mmD : mD$, that is, as $4m : 1$, and conſequently theſe Forces are in the ſame *Ratio*. Hence in particular Caſes the Proportion of the Forces is eaſily known. *Ex. gr.* If $m=\frac{1}{4}$, then $4\ m=1$, and ſo the Centrifugal Force in that Caſe is equal to the Gravity. If $m=\frac{1}{2}$, then $4\ m=2$, and ſo the Proportion of the Forces is as 2. 1. If $m=\frac{1}{4}$, then the Proportion is as 3. 1. If $m=1$, then 'tis as 4. 1. and in like manner for any other ſort of Suppoſition. The Senſe of all which may be expreſs'd in this *Theorem*, That if a Body revolves in a Circle, with a Velocity equal to that acquir'd by a Deſcent thro' a Space, that is to the Diameter of the Circle, as m to Unity; that then the Centrifugal Force ſhall be to that of Gravity, as $4m$ to Unity.

SCHOL. I.

Having hinted a certain *Mechanick Theorem* before concerning the Proportion of the Spaces deſcrib'd by an equable and an uniformly accelerated Motion, in the ſame Time, I think it not improper to ſubjoin a ſhort Illuſtration of it. Let the Quantity x expreſs Time, y Velocity, the greateſt Time d, the greateſt Velocity b; the Two former Quantities being flowing ones, and the latter ſtanding. From the Nature of the uniformly acclerate Motion, the Velocities are proportional to the Times. Therefore $x : y :: d : b$, and $x = \frac{yd}{b}$, and $\dot{x} = \frac{\dot{y}d.}{b}$

Alſo in all Motions whatſoever, the Spaces deſcribed are as the Rectangles under the Velocities and the Times. Therefore the Fluxion of the Space for the accelerate Motion, is $= \dot{y}x$, and that for the equable Motion is $= b\dot{x}$. But the Fluent of $b\dot{x}$ is $= bx = bd$, and the Fluent of $y\dot{x}$ $(= \frac{y\dot{y}d}{b})$ is $= \frac{yyd}{2b} = \frac{bbd}{2b} = \frac{bd}{2}$. Therefore the Space deſcrib'd by the equable Motion in the Time d with the ſtanding Velocity b, is double the Space deſcrib'd by the uniformly Accelerate, in the ſame Time d, with a flowing Velocity, the greateſt of which is equal to b (the Velocity of the uniform Motion.) And from hence to determine the Proportion of the Spaces, deſcrib'd by an equable Motion with the Velocity b, and in the Time d, *Ex. gr.* and an uniformly accelerated one in the Time nd, where n denotes any whole or broken Number. It is (from the Nature of uniformly accelerated Motion, in which the Spaces are as the Squares of the Times) $d^2 : n^2d^2 :: \frac{bd}{2} : \frac{n^2d^2b}{2d^2} = \frac{nnbd}{2} =$ the Space deſcrib'd by the accelerate Motion in the Time nd; the Space deſcrib'd by it in the Time d being $= \frac{bd}{2}$, by what was ſhewn before. So that the Proportion required is that of $bd : \frac{nnbd}{2}$, that is, of $1 : \frac{nn}{2}$, or of $2 : nn$ univerſally; which may be exemplified in particular Caſes at Liberty.

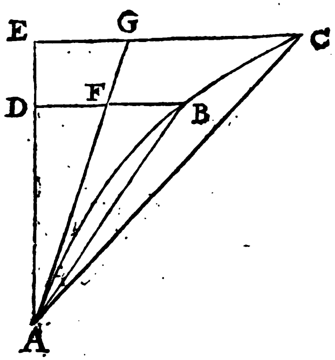
SCHOL. II.

Now, to apply this to the former Compariſon between the Centrifugal Force and that of Gravity. Let the Arch A be deſcribed by the Body revolving in a Circle with an uniform Motion, in the Time d, and the Line k deſcribed by its Deſcent with an uniformly accelerate Motion in the Time nd; according to the latter part of the foregoing *Scholium*. Then

it

is univerſally $A : k :: 2 : nn$, and $A = \frac{2k}{nn}$, from whence $\frac{A^2}{D}$ (the Space deſcrib'd by the Centrifugal Force in the Time d) is $= \frac{4kk}{Dn^4}$. Suppoſe (as before at *Cor.* II.) $k = md$, then $\frac{4kk}{Dn^4} = \frac{4mmDD}{Dn^4} = \frac{4mmD}{n^4}$; and conſequently the Proportion of the Spaces deſcrib'd by the Centrifugal Force in the Time d, and by the Gravity in the Time nd, is as $\frac{4mmD}{n^4} : mD$, that is as $\frac{4m}{n^4} : 1$, or as $4m : n^4$. Of which more general Proportion, that at *Cor.* II. is but a particular Caſe, *viz.* when the Times are equal, or when n is equal to Unity.

Fig. 6.



LEMMA.

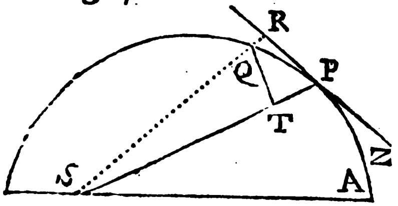
If the Right Line AE (FIG. VI.) *and the Curve AC, given in Poſition, cut one another in the given Angle A; and if the Right Lines DB, EC be ordinately apply'd to the Line AE (in another given Angle) meeting the Curve in the Points in B, C; Then ſuppoſing the Points B, C, to come to the Points A, the Curvilineal Triangles ADB, AEC, ſhall be at laſt in the duplicate* Ratio *of their Homologous Sides.*

FOR drawing the Tangent AFG; upon the Approach of the Points B, C, to A, the *Ratio* of the Tangents AG, AF, the Chords AC, AB, and the Arches AC, AB is a *Ratio* of Equality (by *Lemma* I.) Therefore the Curvilineal Triangles ADB, AEC are at laſt coincident with the Rectilineal Triangles ADB, AEG; but theſe being ſimilar (by the *Hypotheſis*) are in a duplicate *Ratio* of their Homologous Sides (by the *Elements*;) therefore the Curvilineal ones are ſo too.

C o r.

It is evident, that the Spaces which a Body (urged by any regular Force) deſcribes, are in the Beginning of the Motion as the Squares of the Times: For if the Times be expreſs'd by the Lines AD, AE, and the Velocities in thoſe Times by BD, EC, then (from *Mechanicks*) the Spaces ſhall be as the *Area's* ADB, AEC; that is, in the Beginning of the Motion, as the Squares of the Times AD, AE.

Fig. 7.



THEOR. VI.

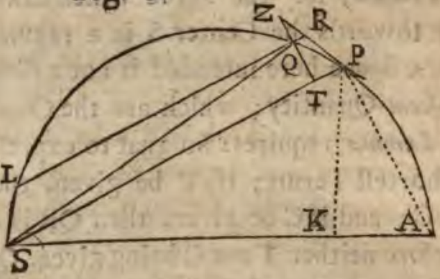
If a Body revolves (FIG. VII.) *about a certain Center S, and deſcribes a Curve APQ; then if ZPR touches the Curve in P, and from any other Points Q be drawn QR parallel to SP, and QT perpendicular to the ſame; the Centripetal Force ſhall be reciprocally as the ſolid* $\frac{SP^{q}\times QT^{q}}{QR}$; *taking the Value of it, not where the Points P and Q are at any finite Diſtance, but at laſt when they come together.*

PUT C for Centripetal Force, and T for Time. Now, in the indefinitely ſmall Figure QRPT, the *Lineola* QR, being the genuine Effect of the Centripetal Force, will be in a given Time proportional to the Force, (by the Second *General Law of Motion.*) But if the Force it ſelf were given, the *Lineola* QR would be as the Square of the Time (by the foregoing *Lemma*) for the Force which draws the Body towards the Center S is a regular one, and the Space here intended is not a *Finite*, but a *Naſcent* Quantity; which are the Conditions that *Lemma* requires: So that to expreſs it in the ſhorteſt Terms; if T be given, then QR is as C; and if C be given, then QR is as T^2; therefore neither T nor C being given, QR is in the *Ratio* compounded of both, that is, QR is as $C \times T^2$, and conſequently C is as $\frac{QR}{T^2}$. But the Time is as the *Area* (by *Theor.* I.) that is, T is as $\frac{1}{2}$ SP × QT (= the *Area* SPQ.) and $\frac{1}{2}$ SP×QT is as SP×QT; ſo that C is as $\frac{QR}{SP^q \times QT^q}$ directly, that is as $\frac{SP^q \times QT^q}{QR}$ Reciprocally. Q.E.D.

COR.

Hence if any Curve be given, and within it a Point, to which (as a Center) the Centripetal Force is directed; we may find the Law of the Centripetal Force, which makes the Body to revolve in the Periphery of that Curve. In order to this, we are to compute the Value of this ſolid $\frac{SP^q \times QT^q}{QR}$, according to the Nature of the Curve: Some Examples of which follow.

Fig. 8.



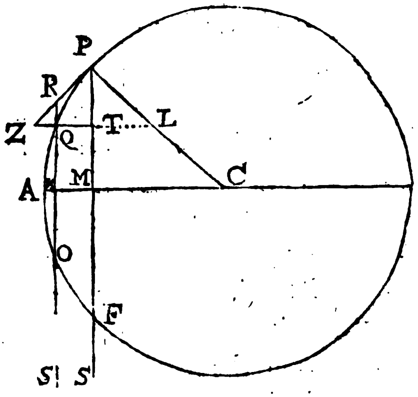
PROB. III.

A Body revolving in the Circumference of a Circle, 'tis requir'd to find the Law of the Centripetal Force tending to a Given Point in the Circumference.

LET P (*FIG.* VII.) be the Place of the Body in the Periphery SQPA, Q the next Place into which it moves, and S the Center to which the Forces are directed. Let PK be perpendicular from P to the Diameter SA, QT perpendicular from Q to SP, LQR parallel to SP, RP a Tangent at P meeting QT, produced in Z. In the Firſt place here, the Triangles ZQR, ZTP, SPA are ſimilar. That the Triangles ZQR, ZTP are ſimilar is plain, becauſe

QR is parallel to the Line SP by the Conſtruction. Again, that the Triangles *Z*TP, SPA are ſimilar, is evident, for the Angle SPA is a Right one from the Nature of the Circle, and the Angle ZTP is a Right one by the Conſtruction; alſo ZP being a Tangent, and SP a Chord, cutting the Circle at P, the Point of Contact, the Angle ZPT is = Angle SAP (ſtanding upon the Arch SQP) from the *Elements*. Therefore the Angle TZP=Angle PSA, and conſequently the Triangles are ſimilar. From whence $ZP^q : ZT^q :: SA^q : SP^q$, and $ZP^q : ZT^q :: RP^q : QT^q$ (becauſe of the proportional Section of the Sides ZP and ZT by the Parallel QR ;) therefore $RP^q : QT^q :: SA^q : SP^q$. But RP being a Tangent, and RQL a Secant, cutting it in R, and the Circle in Q, therefore (from the *Elements*) $RP^q =$ the Rectangle $QR \times RL$, therefore the Analogy ſtands thus $QR \times RL : QT^q :: SA^q : SP^q$; from whence $QT^q = \frac{QR \times RL \times SP^q}{SA^q}$. And becauſe when the Points Q and P come together, the Line RL becomes=SP, therefore putting SP for RL, we have $QT^q = \frac{QR \times SP^3}{SA^q}$. Now (by *Theor.* VI.) the Centripetal Force is reciprocally as $\frac{SP^q \times QT^q}{QR}$, and we have found the Value of QT^q in the proper Terms of the Curve, therefore multiplying this Value of QT^q by $\frac{SP^q}{QR}$, we have $\frac{SP^5}{SA^q}$, which is Reciprocally as the Centripetal Force in this Caſe; that is, becauſe SA, the Diameter, is a given Quantity, the Centripetal Force reciprocally as SP^5, that is the Square-Cube of the Diſtance from the Center S. Q. E. I.

Fig. 9.



P R O B. IV.

A Body revolving in the Circle PQA (Fig. 9.) *'tis requir'd to find the Law of the Centripetal Force tending to a Point S ſo far diſtant, that all the Lines, as PS, RS, &c. drawn to that Point, may be eſteem'd as parallel.*

FROM the Center C draw CA, cutting the Parallels PS, RS, at Right Angles; join the Points C, P, and produce ZT to cut CP in

L. Here ZP is a Tangent, as before, and the Line ZTL at Right Angles to the Parallels PS and RS. Now, the Triangles ZRQ, ZPT, CPM are ſimilar. For ZT cutting the Parallels PS, RS at Right Angles, 'tis plain, that the Triangles ZRQ, ZPT are ſimilar. Alſo, becauſe the Angle ZPC is a Right one, and the Line PT a Perpendicular from the Right Angle to the Baſe ZL; therefore the Triangle ZPT is ſimilar to the Triangle PTL; and ſince the Triangle PTL is ſimilar to PMC; therefore ZPT and PMC are ſimilar. Therefore $ZP^q : ZT^q :: CP^q : PM^q$; but $ZP^q : ZT^q :: RP^q : QT^q$ (from the proportional *Section* of the Sides ZP and ZT by the Parallel RQ;) therefore $RP_q : QT^q :: CP^q : PM^q$. But (from the Nature of the Circle RP being a Tangent, and RQO a Secant, cutting it in the Point R, and the Circle in O) $RP^q = OR \times QR$; from whence $QT^q \left(= \frac{RP^q \times PM^q}{CP^q}\right)$ is $= \frac{OR \times QR \times PM^q}{CP^q}$. But when the Points P and Q meet together, OR becomes $= PF = 2PM$; therefore putting 2PM for OR, we have $QT_q = \frac{2PM^q \times QR}{CP^q}$; and ſo the general *Formula* $\frac{SP^q \times QT^q}{QR}$ is equal $\frac{2PM^q \times SP^q}{CP^q}$. Now, the Point S being at an infinite Diſtance, SP is a ſtanding Quantity, and ſo is CP the *Radius*; ſo that dividing by the ſtanding Quantity

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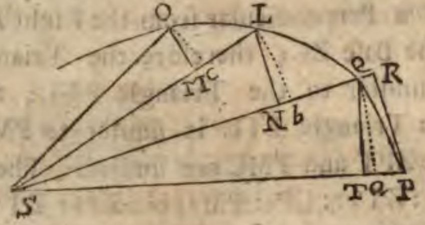
$\frac{2\,SPq}{CPq}$, the Expreſſion amounts to PM'. Therefore the Centripetal Force is reciprocally as the Cube of the Ordinate PM. Q. E. I.

Fig. 10.



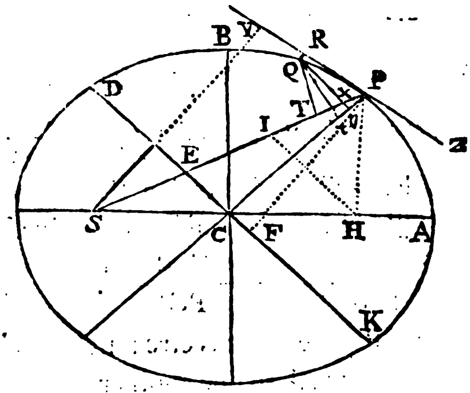
PROB. V.

A Body revolving in the Logarithmick or Proportional Spiral, 'tis requir'd to find the Law of the Centripetal Force tending to the Center of the Spiral.

THE Nature of this Curve (*FIG.*X.) is ſuch, that all the Angles, as SPQ, SQL, SLO, *&c.* (which the Rays make with the Curve or Tangents at thoſe Points) are ſtill of the ſame Magnitude. From whence it follows, that taking all the indefinitely ſmall Angles about the Center S continually equal, that is PSQ=QSL=LSO, *&c.* the Triangles PSQ, QSL, LSO, *&c.*

are ſimilar, and therefore $PS : QS :: QS : SL :: SL : SO$, &c. Conſequently the Rays PS, QS, SL, &c. are in the continual Geometrick Proportion. Therefore ſince $SP : SQ :: SQ : SL$, 'tis alſo $SP : SP - SQ :: SQ : SQ - SL$, that is (deſcribing the indefinitely little Arches Qa, Lb, Oc, upon the Center S) we have $SP : Pa :: SQ : Qb$ or (which amounts to the ſame, letting fall the Perpendiculars QT, LN, OM) 'tis $SP : PT :: SQ : QN$. So that the *Lineolæ* PT, and QN, are always as their reſpective Rays SP and QS. Laſtly, becauſe the indefinitely ſmall Triangles PQT, QLN, &c. are always ſimilar (for the Angles at P and Q are equal from the Nature of the Curve, and thoſe at T and N are Right ones) therefore $PT : QT :: QN : LN$, and ſo PT and QN are always proportional to QT and LN. But before 'twas ſhewn, that PT and QN were always as their reſpective Rays SP and QS; therefore alſo the *Lineolæ* QT and LN are ever as thoſe Rays SP and QS. I thought this little Explication of the moſt common Properties of the Curve, neceſſary to be premis'd, in order to the underſtanding our Great Author's Proceſs in the Invention of the Law of the Centripetal Force. Now, let the indefinitely ſmall Angle PSQ be given or ſuppoſed always of the ſame Bigneſs; then in the Triangle SPR, ſince the Angle at S is always the ſame, and that at P alſo ever the ſame (from the Nature of the Curve) therefore the Angle at R, *viz.* SRP, is ever the ſame. So in the Triangle SQT, ſince the Angle at T is ever a Right one, the Angle SQT ſhall always be the ſame. From hence therefore in the indefinitely ſmall Figure RQTP, the Angles at P, T and R are ſtanding, and ſince the Angle SQT was ſhewn to be ſo, then RQT, the Complement of SQT to Two Right Angles, ſhall be of a ſtanding Magnitude too. Therefore all the Angles of the Figure RQTP, are of a ſtanding Magnitude, and conſequently the Species of the Figure it ſelf is ſtanding, or always given. Therefore the *Ratio* of the Sides QT, QR, *viz.* $\frac{QT}{RQ}$, is a ſtanding Quanity, and ſo $\frac{QT}{QR}$ may be always as 1. Conſequently if $\frac{QT}{QR}$ be as 1, then (multiplying both by QT) $\frac{QT^q}{QR}$ ſhall be always as QT. But (as was ſhewn before) from the Nature of the Curve, QT is as SP; therefore alſo $\frac{QT^q}{QR}$ ſhall be as SP, therefore $\frac{QT^q \times SP^q}{QR}$ (which is reciprocally as the Centripetal Force) ſhall be as SP^3. Therefore the Centripetal Force in this Curve is Reciprocally as the Cube of the Diſtance from the Center S. Q. E. I.

Fig. 11.



PROB. VI.

A Body revolving in an Ellipſe, 'tis requir'd to find the Law of the Centripetal Force of a Body tending to the Center of that Ellipſe.

LET CA, CB, (*FIG.* XI.) be the *Semiaxes* of the Ellipſe; GP, DK conjugate Diameters; PF, Qt Perpendiculars to thoſe Diameters; Qv an ordinate Applicate to the Diameter GP; and compleat the Parallelogram $QvRP$: The Triangles Qtv, PCF are ſimilar, for the Angles at t and F are Right ones by the *Hypotheſis*; but Qv being parallel to CF the Angle $Qvt = FCP$; therefore $Qt^q : Qv^q :: PF^q : PC^q$, and $Qt^q = \frac{Qv^q \times PF^q}{PC^q}$. But (from *Conick Sect.*) $Qv^q : Pv \times vG :: CD^q : CP^q$; therefore Pv

=

$=\frac{Qv^q\times CP^q}{vG\times CD^q}$; but $Pv=QR$, therefore we have this Value of QR. Therefore in the general Expreſſion for the Centripetal Force, *viz.* $\frac{PC^q\times Qt^q}{QR}$ we ſhall have (by ſubſtituting the Values of Qt^q and QR, but now found) this Value, *viz.* $\frac{Qv^q\times PF^q\times vG\times CD^q}{Qv^q\times PC^q}=\frac{PF^q\times CD^q\times vG}{PC^q}$. But (from *Conicks*) all Porallelograms circumſcrib'd about an Ellipſe are equal to one another; therefore $CD\times PF=BC\times CA$, and $CD^q\times PF^q=BC^q\times CA^q$, and ſubſtituting $BC^q\times CA^q$ in the Room of $CD^q\times PF^q$, the Expreſſion comes to $\frac{BC^q\times CA^q\times vG}{PC^q}$. But when the Points P and Q come together, then vG is $=2PC$, ſo that then the Expreſſion amounts to this, *viz.* $\frac{BC^q\times CA^q\times 2PC}{PC^q}$. But $2BC^q\times CA^q$ is a ſtanding Quantity, therefore the Expreſſion comes to $\frac{PC}{PC^q}$, *i. e.* $\frac{1}{PC}$; ſo that the Centripetal Force is reciprocally as $\frac{1}{PC}$, *i. e.* directly as PC, the Diſtance from the Center. Q. E. I.

Cor.

Cor. I.

Hence it follows, that the Centripetal Force is greateſt at the greateſt Diſtance from the Center. 'Tis greater (*Ex.gr.*) in the Extremity of the longer Axe,than in any other Point in the Curve of the Ellipſe, between that and the Extremity of the ſhorter Axe.

Cor. II.

At equal Diſtances from either of the principal Vertices, the Centripetal Forces are equal. For the Diameters or Diſtances from the Center (from the Nature of the *Ellipſe*) are ſo in this Caſe.

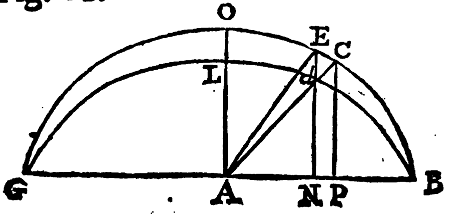
Cor. III.

Any Two equal Diameters being taken thus, one of one Side the principal Vertex, and the other of the other Side; the Conjugates of theſe equal Diameters being equal alſo, it's evident, that the Sum of the Centripetal Forces in the Extremities of one of theſe Diameters, and its Conjugate, ſhall be equal to the Sum of the Forces in the Extremities of the other Diameter, and its Conjugate. *Ex.gr.* Let A be any Diameter, *a* its Conjugate; B a Diameter taken on the other Side at an equal Diſtance from the principal Vertex, and conſequently equal to A; and b its Conjugate, which is therefore equal to a. Let F = the Centripetal Force in A, f = Force in a, φ=Force in B, and π=Force in b. Now I ſay, that becauſe $A+a=B+b$, therefore $F+f=\varphi+\pi$. For (by the Law of the Centripetal Force found in the laſt *Problem.*) 'Tis $F : f :: A : a$, and $\varphi : \pi :: B : b$, from whence ſince $A+a=B+b$, alſo $F+f$ ſhall $=\varphi+\pi$, that is, the Sums of the Centripetal Forces in the Extremities of each Pair of theſe Conjugate Diameters ſhall be equal.

COR. IV.

Therefore alſo $F-\varphi$, or $\varphi-F$ is $=\pi-f$, or $f-\pi$; that is, the Difference of the Forces in the Extremities of the Diameters A and B, is equal to the Difference of the Forces in the Extremities of their Conjugates a and b.

Fig. 12.



COR. V.

In Ellipſes that have the longer Axe common, drawing any Diameter, as AC (*FIG.* XII.) cutting the Curves in C and d; the Centripetal Force in d ſhall be to the Centripetal Force in C, as $Ad : AC$. Let the Force in B be put $= F$, Force in $d = f$, Force in $C = \varphi$. Then the *Ratio* of the Force in d to the Force in C is compounded of the *Rationes* of the Force in d to that in B, and of the Force in B to that in C. That is, $\frac{f}{\varphi} = \frac{f}{F} \times \frac{F}{\varphi}$. But $\frac{f}{F} = \frac{Ad}{AB}$, and $\frac{F}{\varphi} = \frac{AB}{AC}$ (from the Law of the Centripetal Force in each Ellipſe) therefore $\frac{f}{\varphi} = \frac{Ad}{AB} \times \frac{AB}{AC} = \frac{Ad}{AC}$; ſo that $f : \varphi :: Ad : AC$. In Two Ellipſes therefore that have the ſame Tranſverſe Axe, the Centripetal Force (which tends to their common Center) is greateſt in the exterior Ellipſe. And in the Extremities of the ſhorter Axes, *viz.* in the Points L, O, the Centripetal Forces will be as thoſe Axes, *viz.* as $AL : AO$; or (which is all one) drawing any Ordinate, as EdN cutting the Ellipſes in E and d, the Forces in the Two Ellipſes will be as $dN : EN$; for 'tis $AL : AO :: dN : EN$, from the Nature of the Ellipſe.

COR. VI.

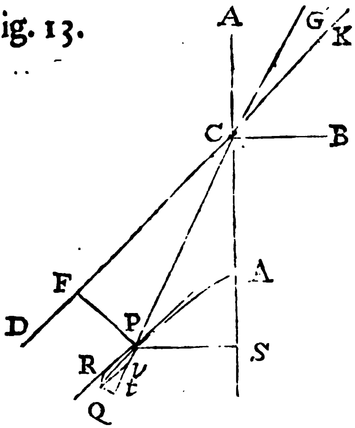
Suppoſing all as in the laſt *Corollary* (with its *Fig.*) from the Point C in the exterior Ellipſe, draw the Ordinate CP. Then ſhall the *Ratio* of the Forces in the Points O and L, be to the *Ratio* of the Forces in the Points C and *d*, as $\sqrt{GN \times NB} : \sqrt{GP \times PB}$. For the *Ratio* of the Forces in the Points O and L is $= \frac{AO}{AL} = \frac{EN}{dN}$, by what was ſhewn at the latter End of the preceding *Corollary*. Alſo the *Ratio* of the Forces in C and *d*, is $= \frac{AC}{Ad}$, by what was ſhewn in the former Part of the foregoing *Corollary*. But becauſe *d*N and CP are perpendicular to GB (by the *Hypotheſis*) therefore from the ſimilar Triangles ACP, A*d*N, we have $\frac{AC}{Ad} = \frac{CP}{dN}$, from whence $\frac{CP}{dN}$ is = the *Ratio* of the Forces in the Points C and *d*. Therefore the *Ratio* of the Forces in the Points O and L, is to the *Ratio* of the Forces in the Points C and *d*, as $\frac{EN}{dN} : \frac{CP}{dN}$. But $\frac{EN}{dN} : \frac{CP}{dN} :: EN : CP$, and $ENq : CPq :: GN \times NB : GP \times PB$ (from the Nature of the *Ellipſe*)

lipſe;) conſequently EN : CP :: $\sqrt{GN \times NB} : \sqrt{GP \times PB}$, from whence the Aſſertion is clear.

SCHOL.

The *Great Author* has briefly ſhewn how this Matter may be carried from the Ellipſe to the Hyperbola and Parabola, and how the Law of the Forces with Reſpect to thoſe Curves may be alſo diſcover'd. And here by his neat Application of it to the Parabola, he brings out a celebrated *Theorem* of *Galilæus*. But becauſe the young Beginners in theſe Things may poſſibly deſire a more particular Explication of it, upon their Account I ſhall make the Application to theſe Two Curves diſtinctly.

Fig. 13.



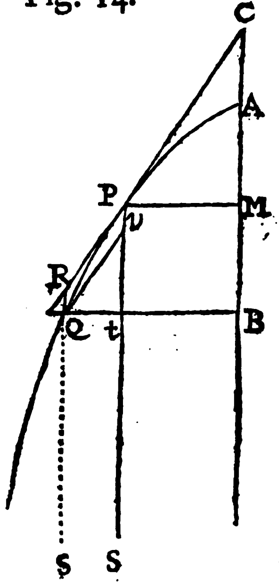
PROB. VII.

If a Body moves in the Curve of an Hyperbola, 'tis requir'd to find the Law of the Force tending to the Center of the Hyperbola.

'TIS evident here in the Firſt place, that the Force tending to the Center, is moſt properly a Centrifugal Force, for the Body ſtill goes farther and farther from that Point to which the Forces are directed. Let (*FIG.* XIII.) C be the Center of the Hyperbola, CP*t* a Diameter, CD its Conjugate, RP a Tangent at P, Q*v* an Ordinate to CP, Q*t* and PF Perpendiculars to the Diameters CP and CD reſpectively, and RQ parallel to P*v*. In a Word, as all the Lines drawn here are Analogous to thoſe in the Scheme for the correſpondent Caſe in the Ellipſe;

lipſe; ſo from the Nature of theſe Curves will all the Reaſonings here proceed after the ſame Manner as there. For here the Triangles PCF and Qvt are ſimilar, becauſe the Angles at F and t are Right Ones by the Hypotheſis, and the Angle $Qvt = PCF$, becauſe Qv is parallel to the Conjugate DC. From hence $Qt^q = \frac{Qv^q \times PF^q}{PC^q}$, as before. Alſo (from *Conicks*) $Qv^q : Gv \times Pv :: CD^q : CP^q$, therefore $Pv = \frac{Qv^q \times CP^q}{Gv \times CD^q} = QR$. From whence by Subſtitution of theſe Values of Qt^q and QR in the general Expreſſion, *viz.* $\frac{Qt^q \times PC^q}{QR}$, it comes to $\frac{PF^q \times CD^q \times vG}{PC^q}$, altogether as in the Ellipſe. Then ſubſtituting $BC^q \times CA^q$ for $PF^q \times CD^q$ (becauſe $BC \times CA = CD \times PF$, from *Conicks*) and 2 PC for vG, (when the Points P and Q come together;) Laſtly, Dividing by the ſtanding Quantity $2BC^q \times CA^q$, the Expreſſion comes to $\frac{1}{PC}$. Therefore the Force, with reſpect to the Point C, is reciprocally as $\frac{1}{PC}$, that is, directly as PC, the Diſtance from the Center. Q. E. I.

Fig. 14.



PROB. VIII.

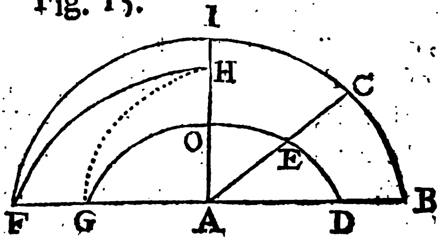
If a Body moves in a Curve of a Parabola, 'tis requir'd to find the Law of the Centripetal Force tending to a Point infinitely diſtant.

LET AB (*FIG.* XIV.) be the Axe of the Parabola, RP a Tangent at P meeting the Axe in C, PM an Ordinate to the Axe, QS and PC Lines tending to the infinitely diſtant Center, and therefore parallel, and conſequently Diameters of the Parabola; Q*v* an

Ordinate to PS, Qt perpendicular to PS; Laſtly, Let QS be produced to meet the Tangent in R. Now, becauſe of the ſimilar Triangles PCM, Qvt, 'tis PC : PM :: $Qv : Qt$, and ſo $Qt^q = \frac{PM_q \times Qv^q}{PC^q}$. Let the Parameter belonging to the Vertex P be put $=R$, and the Parameter of the Axe $=r$, the Abſciſs $AM=x$ Then, from the Nature of the Parabola, we have theſe following Equalities; *viz.* $Qv^q = R \times Pv$, $PM^q = rx$, $CM^q = 4x^2$, $PC^q = rx + 4xx$ (becauſe of the Rectangular Triangle PCM) $R = r + 4x$ (that is, the Parameter of the Vertex P is $=$ the Parameter of the Axe + Quadruple the Abſciſs AM:) Theſe Things are plain from the *Conick Elements.* Now ſubſtituting theſe Values in the Expreſſion of Qt^q, we find $Qt^q = \frac{rx \times Qv^q}{rx+4xx} = \frac{rx \times Pv \times R}{rx+4xx} = \frac{rx \times Pv \times \overline{r+4x}}{rx+4xx} = \frac{Pv \times \overline{rrx+4rxx}}{rx+4xx} = Pv \times r$ (dixiding by $rx+4xx$) ſo that $Qt^q = Pv \times r$. Therefore the general Expreſſion of the Centripetal Force, *viz.* $\frac{PC^q \times Qt^q}{QR}$, comes to $\frac{PC^q \times Pv \times r}{QR}$, which is (becauſe $QR = Pv$ from Parallels) $= PC^q \times r$. But r, the Parameter of the Axe, is a ſtanding Quantity, and becauſe PC is infinite, therefore PC^q is ever a ſtanding Quantity, and conſequently the whole Expreſſion is a ſtanding Quantity, and therefore the Centripetal Force (which is reciprocally as PCq×*r*) is in this Caſe equable. And this is *Galilæus*'s *Theorem*. Q. E. I.

Before we proceed to conſider the Law of the Centripetal Force, when it tends to the *Foci* of the *Conick Sections*, 'tis proper to ſubjoin thoſe other Propoſitions that relate to, and depend upon this preſent Law, in which the Force tends to the Center.

Fig. 15.



LEMMA.

Any Two Ellipſes, as FIB, GOD, (FIG. XV.) *having a common Center A, and the Tranſverſe Axes FB, GD, lying in the ſame Right Line; the Centripetal Forces (tending to the common Center A) ſhall be in the Points F and G, as the Diſtances of thoſe Points from the Center,* viz. *as FA : GA, that is, as the Semitranſverſe Axes.*

FOR upon the ſame Tranſverſe FB of the outward Ellipſe, one may ſtill deſcribe another Ellipſe, as FH, whoſe Semi-conjugate

AH ſhall be equal to AG, the Semitranſverſe of the inward Ellipſe. Now in the Ellipſe FH, the Force in F is to that in H, as FA : HA (by what has been ſhewn.) Alſo the Points H and G being equally diſtant from the Center A, the Forces in H and G are equal ; therefore the Force in F in the Ellipſe FH, is to the Force in G in the Ellipſe GO, as FG: AG (which is =AH :) But the Force in F in the Ellipſe FH, is equal to the Force in F in the Ellipſe FI, ſince the Point F is common to them both. Therefore the Force in F in the Ellipſe FI, is to the Force in G in the Ellipſe GO, as FG : AG. Q. E. D.

THEOR. VII.

In all ſimilar Ellipſes, the Times of the Periodick Revolutions (perform'd about the ſame Center) are equal.

LET the Ellipſes FIB, GOD, (*FIG.* XV. foregoing) be conceiv'd to be ſimilar ; then drawing any Diameter as AC, cutting the interior Ellipſe in E, it might eaſily be demonſtrated (from the ſuppoſed Similarity of the *Figures*) that AD : AE :: AB : AC, or that AO : AE : : AI : AC; and ſo that any other correſpondent Diameters are ſtill in the Proportion of the Axes. Now (the Centripetal Forces in both Ellipſes tending to the common Center A) the Force in B ſhall be to that in D, as AB: AD (by the foregoing *Lemma* :) Therefore alſo the Force in C ſhall be to that in E, as AC : AE, that is, as AB: AD; and ſo drawing any other Diameter, the Centripetal Forces in the Two Ellipſes ſhall ſtill be in the ſame conſtant *Ratio* of the Semiaxes. But if Two Circles were deſcrib'd upon the Center A with the *Radii* AB, AD, the Periodick Times in thoſe Circles wou'd be equal; for the Times are equal(as was ſhewn in the *Corollaries* of *Theor*.IV.) when the Centripetal Forces are directly as the Rays, as here they are. And therefore ſince the Forces are in the ſame *Ratio* of theſe *Radii* AB and AD, every where in the Two Ellipſes, the Periodick Times in them ſhall alſo be equal. Q. E. D.

SCHOL.

In the Demonſtration foregoing, 'twas aſſerted, that the Centripetal Force in C is to that in E, as AC to AE; which is thus ſhewn. The Force in C is to the Force in B, as AC: AB, and the Force in E to that in D, as AE : AD (from the general Law diſcover'd at *Prob.* VI.) But, from the Similarity of the *Figures*, 'tis AC : AB :: AE : AD, therefore Force in C to Force in B, as Force in E to Force in D; and alternately, Force in C to Force in E, as Force in

B to Force in D; but the Force in B and D are (by the foregoing *Lemma*) as AB : AD; that is, as AC, AE, therefore the Forces in C and E are alſo in the ſame *Ratio* of AC : AE.

LEMMA I.

Putting the Parameter of the longer Axe of the Ellipſe = L (FIG. . *of* Prob. VI.) *then ſhall* $L = \frac{Qt^q \times PC^3}{QR \times PC^3}$ (*taking this Quantity where the Points P and Q come together.*

FOR it was found at that *Prob.* that $\frac{Qt^q \times PC^q}{QR}$ was $= \frac{2BC^q \times CA^q}{PC}$. But (from *Conicks*) the Rectangle under the Parameter and the Semi-tranſverſe, is equal to twice the Square of the Semi-conjugate, that is, $L \times CA = 2BC^q$, from whence $BC^q = \frac{L \times CA}{2}$, and ſo $\frac{L \times CA^3}{PC} = \frac{2BC^q \times CA^q}{PC}$ $= \frac{Qt^q \times PC^q}{QR}$. Therefore (by Reduction) $L = \frac{Qt^q \times PC^3}{QR \times CA^3}$. Q. E. D.

LEMMA.

LEMMA II.

Suppoſing all as in the foregoing Lemma, *(for the Centripetal Force tending to the Center of the Ellipſe) I ſay, that the* Area *deſcrib'd in a given Particle of Time, ſhall be expreſs'd by this ſtanding Quantity* $L^{\frac{1}{2}} \times CA^{\frac{3}{2}}$.

FOR by the preceding *Lemma*, $L = \frac{Qtq \times PC^3}{QR \times CA^3}$, therefore $Qtq \times PC^3 = L \times QR \times CA^3$. But in a given Time, the *Lineola* QR is as the generating Centripetal Force (by the Second *General Law of Motion)* that is (in this Caſe) as PC, the Diſtance from the Center. Therefore ſince QR is as PC, then $Qtq \times PC^3$ ſhall be as $L \times PC \times CA^3$, and (dividing by PC) $Qtq \times PCq$ ſhall be as $L \times CA^3$, and $Qt \times PC$ as $L^{\frac{1}{2}} \times CA^{\frac{3}{2}}$. But $Qt \times PC$ is as the *Area* deſrib'd in a given Time. Therefore this *Area* is as the Quantity $L^{\frac{1}{2}} \times CA^{\frac{3}{2}}$. Q. E. D.

COR.

Putting L, *l*, the Parameters, and 2D, 2*d*, the Conjugate or ſhorter Axes of Two Ellipſes that have the ſame common Tranſverſe 2CA; then the *Area's* deſcrib'd in theſe Ellipſes in a given Time will be as the Conjugates. For the

Area's will be as $L^{\frac{1}{2}} \times CA^{\frac{3}{2}} : l^{\frac{1}{2}} \times CA^{\frac{3}{2}}$, that is (dividing by CA) as $L^{\frac{1}{2}} \times CA^{\frac{1}{2}} : l^{\frac{1}{2}} \times CA^{\frac{1}{2}}$, that is, as 2 D : 2 *d*; for (from *Conicks*) $L \times CA =$ and $2D^2$, $l \times CA = 2d^2$.

THEOR. VIII.

The Times of the Periodick Rovolutions (perform'd about the ſame Center) are equal in all Ellipſes that have the ſame Tranſverſe Axe.

LET A be the *Area* of one Ellipſe, T the Periodick Time, D the ſhorter Axe, N a Particle of the *Area* deſcrib'd in a given Time. Let *a* be the *Area* of another Ellipſe, *t* the Periodick Time, *d* the ſhorter Axe, *n* a Particle of the *Area* deſcrib'd in the ſame given Time; and let B be the common Tranſverſe Axe in both theſe Ellipſes. Now, the *Area's* of Two Ellipſes are in the *Ratio* compounded of the Periodick Times, and the Particles of the *Area's* deſcrib'd together in a given Time. For the Periodick Time being the compleat *Number* of thoſe Particles of the *Area*, that altogether compoſe the whole *Area* of the Figure; hence if thoſe Particles are equal, the *Area's* would be as the Periodick Times, or if the Times were equal, the *Area's* would be as thoſe Particles deſcrib'd in a given Time; and therefore,

THEOR. X.

If a Body revolves in an Ellipſe, and the Centripetal Force tends to the Center of the ſame; the Velocity in any Point as P (FIG. XV. Prob.) *ſhall be expreſs'd by this Quantity* $\frac{L^{\frac{1}{2}}\times CA^{\frac{3}{2}}}{PF}$, *or (which is all one) it ſhall be in the Subduplicate* Ratio *of the Quantity* $\frac{PF^{q}}{L\times CA}$, *Reciprocally (the Line* PF *being perpendicular from* P, *to the Diameter* CF, *which is a Canjugate to* PC.)

FOR the Velocity in a given Time ſhall be expreſs'd by the little Arch PQ deſcrib'd in that Time; that is (ſince the little Arch and its Tangent are at laſt coincident) by the Tangent PC, or becauſe of the Parallelogram QRPv) by the *Lineola* Qv. But the Triangles Q$v\,t$, PCF being ſimilar, the *Leneola* Qv = $\frac{PC\times Qt}{PF}$; alſo PC×Qt is as the *Area* deſcrib'd in a given Time. But (by *Lemma* II.) the *Area* deſcrib'd in a given Time is expreſs'd by $L^{\frac{1}{2}}\times CA^{\frac{3}{2}}$, therefore Q$v$ is as $\frac{L^{\frac{1}{2}}\times CA^{\frac{3}{2}}}{PF}$; that is, the Velocity is in the ſubduplicate *Ratio* of $\frac{\text{PFq}}{\text{L}\times\text{CA}}$ Reciprocally. Q. E. D.

That the *Corollaries* may be more eaſily apprehended, I ſhall premiſe ſome preparatory Interpretations of this general Expreſſion of the Velocity. In order to which, let 2 D. 2 *d* be the ſhorter Axes, L. *l*. the Parameters (of the longer Axes of any Two Ellipſes) P. *p*. the Perpendiculars from the Centers to the Tangents; or, which is all one, the Lines PF, mention'd juſt now: The general Expreſſion now, *viz.* $\frac{\text{L}^{\frac{1}{2}}\times\text{BA}^{\frac{1}{2}}}{\text{PF}}$, in theſe more compendious Terms comes to $\frac{\text{L}^{\frac{1}{2}}\times\text{B}^{\frac{1}{2}}}{\text{P}}$, or $\frac{l^{\frac{1}{2}}\times b^{\frac{1}{2}}}{p}$, for one and the other Ellipſe. And to determine this to particular Points of the Curve (*Ex.gr.*) in the Extremities of the longer and ſhorter Axes, we need only conſider, that in thoſe Extremities the Perpendiculars P. *p*. are coincident with the half longer and ſhorter Axes themſelves. Therefore,

1. In the Extremity of the ſhorter Axe, the Expreſſion comes to $\sqrt{2}\times\text{B}$. For in this Caſe P is $=$ D $=$ to half the ſhorter Axe $=\sqrt{\frac{\text{L}\times\text{B}}{2}} = \frac{\text{L}^{\frac{1}{2}}\times\text{B}^{\frac{1}{2}}}{\sqrt{2}}$, as is plain from the Nature of the

Ellipſe. Therefore $\frac{L^{\frac{1}{2}}\times B^{\frac{1}{2}}}{P}$ $\left(=\frac{L^{\frac{1}{2}}\times B^{\frac{1}{2}}}{D}\right)$ is $=$ $\frac{\sqrt{2}\times L^{\frac{1}{2}}\times B^{\frac{1}{2}}}{L^{\frac{1}{2}}\times B^{\frac{1}{2}}}=\sqrt{2}\times B$.

2. In the Extremity of the longer Axe, the Expreſſion comes to $L^{\frac{1}{2}}\times B^{\frac{1}{2}}$. For here in this Caſe, P is $=$ B $=$ to half the longer Axe.

3. For a Circle whoſe *Radius* is $=\frac{1}{2}$ the ſhorter Axe (putting R the *Radius*) the Expreſſion of the Velocity comes to $\sqrt{2}\times R$ in the Terms of the Circle; and to $L^{\frac{1}{2}}\times B^{\frac{1}{2}}$ in the Terms of the Ellipſe. The former part is clear: For in the Circle L is $=2R$, $B=R$, and $P=R$, therefore $\frac{L^{\frac{1}{2}}\times B^{\frac{1}{2}}}{P}$ is $=\sqrt{2}\times R$. The latter part appears, in that R is $=$ D by the *Hypotheſis*, that is, $R=\frac{L^{\frac{1}{2}}\times B^{\frac{1}{2}}}{\sqrt{2}}$, and ſo $R\times\sqrt{2}=L^{\frac{1}{2}}\times B^{\frac{1}{2}}$ in the Terms of the Ellipſe.

4. For a Circle whoſe *Radius* is $\frac{1}{2}$ the longer Axe, the Expreſſion of the Velocity comes (as before) to $\sqrt{2}\times R$ in the Terms of the Circle; and to $\sqrt{2}\times B$ in the Terms of the Ellipſe. For in this Caſe R is $=$B, and ſo $\sqrt{2}\times R$ is $=\sqrt{2}\times B$. Now from hence the following *Corollaries* will appear to be very clear.

Cor.

COR. I.

In ſimilar Ellipſes, and in Points ſimilarly poſited to the principal Vertexes, the Velocities are every where in ſome ſtanding *Ratio* of the longer or ſhorter Axes, or the Parameters.

COR. II.

In Ellipſes that have the ſame Tranſverſe Axe, the Velocities are as the ſquare Roots of the Parameters directly, and the Perpendiculars from the common Center to the Tangents inverſly. For the Expreſſions are $\frac{L^{\frac{1}{2}}\times B^{\frac{1}{2}}}{P}$, and $\frac{l^{\frac{1}{2}}\times b^{\frac{1}{2}}}{p}$; therefore if $B = b$, the Velocities in any Two ſuch Ellipſes are as $\frac{L^{\frac{1}{2}}}{P} : \frac{l^{\frac{1}{2}}}{p}$.

COR. III.

The Tranſverſe Axe being common, the Velocities in the Two Ellipſes in the Extremity of the Tranſverſe Axe, are as the ſhorter Axes directly. For in this Caſe B is $= b$, and the Perpendiculars P. *p*. do both coincide with B or *b*; therefore the Velocities are as $L^{\frac{1}{2}} : l^{\frac{1}{2}}$. But becauſe $B = b$, therefore $L^{\frac{1}{2}} : l^{\frac{1}{2}} :: L^{\frac{1}{2}}\times B^{\frac{1}{2}} : l^{\frac{1}{2}}\times b^{\frac{1}{2}}$, which Exppreſſions are as the ſhorter Axes; therefore, &c.

COR. IV.

Suppoſing all as before, the Velocities in the Extremities of the ſhorter Axes are equal: For P and p are here, coincident, with D and d, alſo, by the *Hypotheſis*, B is $= b$, therefore the Expreſſions of the Velocities in the Two Ellipſes amount to $\frac{L^{\frac{1}{2}}}{D}$, and $\frac{l^{\frac{1}{2}}}{d}$; that is (putting in the Values of D and d) to $\frac{\sqrt{2\times L^{\frac{1}{2}}\times B^{\frac{1}{2}}}}{L^{\frac{1}{2}}}$, and $\frac{\sqrt{2\times l^{\frac{1}{2}}\times b^{\frac{1}{2}}}}{l^{\frac{1}{2}}}$, which are equal, becauſe $B = b$.

COR. V.

In one and the ſame Ellipſe, the Velocities are Reciprocally as the Perpendiculars from the Center, to the Tangents at any Points where the Body is ſuppoſed to be. For the Expreſſion $\frac{L^{\frac{1}{2}}\times B^{\frac{1}{2}}}{P}$ (the Numerator being a ſtanding Quantity, and conſequently made equal to Unity) comes to $\frac{1}{P}$. So that the Velocity in the Extremity of the ſhorter Axe, is to the Velocity in the Extremity of the longer Axe, as the longer Axe is to the ſhorter.

COR.

Cor. VI.

The Velocity in the Extremity of the ſhorter Axe, is to the Velocity in a Circle at the ſame Diſtance, as the longer Axe is to the ſhorter. For (by the firſt *Preliminary* foregoing) the Expreſſion of the Velocity for the Extremity of the ſhorter Axe is $\sqrt{2 \times B}$; and (by *Prelim.* III.) that for a Circle whoſe *Radius* is $= \frac{1}{2}$ the ſhorter Axe, is $\sqrt{L \times B}$. But $\sqrt{2 \times B} : \sqrt{L \times B} :: B : \sqrt{\frac{L \times B}{2}}$; which Proportion is manifeſt, and the Two latter Terms are the half longer and ſhorter Axes. Therefore, *&c.*

Cor. VII.

The Velocity in the Extremities of the longer Axe, is to the Velocity in a Circle at the ſame Diſtance, as the ſhorter Axe is to the longer. For (by the 2d *Prelim*) the Expreſſion for the Extremity of the longer Axe is $\sqrt{L \times B}$, and (by *Prelim.* IV.) that for a Circle whoſe *Radius* is = half the longer Axe, comes to $\sqrt{2 \times B}$, the former of which Expreſſions is evidently to the latter, as the ſhorter Axe to the longer.

COR. VIII.

Comparing the Accounts of the Two laſt *Corollaries*, it will be found, that the Expreſſions of the Velocities in the Ellipſe at the Extremity of the ſhorter Axe, and in a Circle whoſe *Radius* is = half the longer Axe, are the ſame; and likewiſe of the Velocity in the Ellipſe at the Extremity of the longer Axe, and that in a Circle whoſe *Radius* is = half the ſhorter Axe, are the ſame, *viz.* for the Two former $\sqrt{2\times B}$, and for the Two latter $\sqrt{L\times B}$; ſo that the Velocities in theſe Circles, and theſe Points of the Ellipſe are equal.

COR. IX.

Two Ellipſes having a common Center, and their longer Axes lying in the ſame Right Line; if they are ſo proportion'd to each other, that the Parameters of the longer Axes be reciprocally as thoſe Axes; then the Velocities in the Extremity of the ſhorter Axe ſhall be in each Ellipſe, as the longer Axes directly. It muſt be obſerv'd here, that theſe Ellipſes thus proportion'd to one another ſhall have the ſhorter Axe common to them both. For, by the Suppoſition, 'tis $L : l :: b : B$, from whence $L\times B = l\times b$, and ſo $\sqrt{\frac{L\times B}{2}}=\sqrt{\frac{l\times b}{2}}$, or $D=d$.

$D = d$. Now the Expreſſions of the Velocities in the Extremity of the ſhorter Axe, for each Ellipſe are $\frac{L^{\frac{1}{2}}\times B^{\frac{3}{2}}}{D}$, and $\frac{l^{\frac{1}{2}}\times b^{\frac{3}{2}}}{d}$, that is (putting in the Value of D and *d*) $\sqrt{2}\times B$, and $\sqrt{2}\times b$, which are as B and *b*. Therefore, *&c.*

COR. X.

Suppoſing all as in the laſt *Corollary*, the Velocities in the Extremities of the longer Axes are equal to one another: For in this Caſe the Expreſſions amount to $\frac{L^{\frac{1}{2}}\times B^{\frac{3}{2}}}{B}$, and $\frac{l^{\frac{1}{2}}\times b^{\frac{3}{2}}}{b}$, that is, to $L^{\frac{1}{2}}\times B^{\frac{1}{2}}$, and $l^{\frac{1}{2}}\times b^{\frac{1}{2}}$, or $\sqrt{L\times B}$ and $\sqrt{l\times b}$; but theſe Expreſſions are equal, becauſe $L\times B$ is $= l\times b$, as was ſhewn in the laſt *Corollary*. Therefore, *&c.*

COR. XI.

Comparing *Corolls.* the 3d and 4th with *Corolls.* the 9th and 10th, we may obſerve a Parallel not unworthy Conſideration. For (by *Cor.* III.) the longer Axe being common, the Velocities in the Extremities of the longer Axe are as the ſhorter Axes directly; and (by *Cor.* IX.) the ſhorter Axe being common, the Velocities in the Extremity of the ſhorter Axe are as the longer Axes directly. Again, (by *Cor.* IV.) ſuppoſing what was ſuppoſed in *Cor.* III. the Velocities in the Extremities of the ſhorter Axes are equal; and (by *Cor.* X.) ſuppoſing what was ſuppoſed in *Cor.* IX. the Velocities in the Extremities of the longer Axes are alſo equal to one another.

Cor. XII.

If Two Ellipſes be ſo proportion'd to one another, that the ſhorter Axe of the one be equal to the longer Axe of the other, then the Velocity in the Extremity of the longer Axe of the one Ellipſe ſhall be equal to the Velocity in the Extremity of the ſhorter Axe of the other. For the Expreſſion of the Velocity in the Extremity of the longer Axe of one Ellipſe is $\frac{L^{\frac{1}{2}}\times B^{\frac{3}{2}}}{B}=L^{\frac{1}{2}}\times B^{\frac{1}{2}}$; and that for the Velocity in the Extremity of the ſhorter Axe of the other, is $\frac{l^{\frac{1}{2}}\times b^{\frac{3}{2}}}{d}=\frac{\sqrt{2}\times l^{\frac{1}{2}}\times b^{\frac{3}{2}}}{l^{\frac{1}{2}}\times b^{\frac{1}{2}}}=\sqrt{2}\times b$.

Now, if $L\times B=2bb$, or $\sqrt{L\times B}=\sqrt{2}\times b$, or $\sqrt{\frac{L\times B}{2}}$ (the Semi-conjugate of the one) be $=b$ (the Semi-tranſverſe of the other) then 'tis plain, that theſe Expreſſions of the Velocities are equal. Or we may ſay it will be ſo, if it be $L : 2b :: b : B$; if the Parameter of one Ellipſe be to the Tranſverſe of the other, as the

Semi-tranſverſe of that other to the Semi-tranſverſe of the firſt.

C O R. XIII.

If the Center of the Ellipſe C be imagin'd to remove to an infinite Diſtance, and ſo the Ellipſe to change into a Parabola, a Body ſhall move in the ſame with an equable Velocity. For in this Caſe all the Quantities that enter the general Expreſſion, *viz.* $\frac{L^{\frac{1}{2}} \times B^{\frac{3}{2}}}{P}$, are ſtanding ones, the Quantities B and P being both infinite, and L being the Parameter of the Axe. And here alſo (*mutatis mutandis*) one may apply what has been ſaid in the *Corollaries* foregoing of the true Ellipſe to the Parabola, or Ellipſe whoſe Center is at an infinite Diſtance.

C O R. XIV.

And if this Parabola (by changing the Inclination of the Plane cutting the Cone) degenerates into an Hyperbola, the Body ſhall move in the ſame with a Velocity, which ſhall be reciprocally as the Perpendicular from the Center to the Tangent, as was ſhewn for the Ellipſe at *Cor.* V.

I proceed now to the Conſideration of the Law of the Centripetal Forces and Velocities of Bodies, when thoſe Forces are directed to the *Foci* of the *Conick Sections.*

PROB. IX.

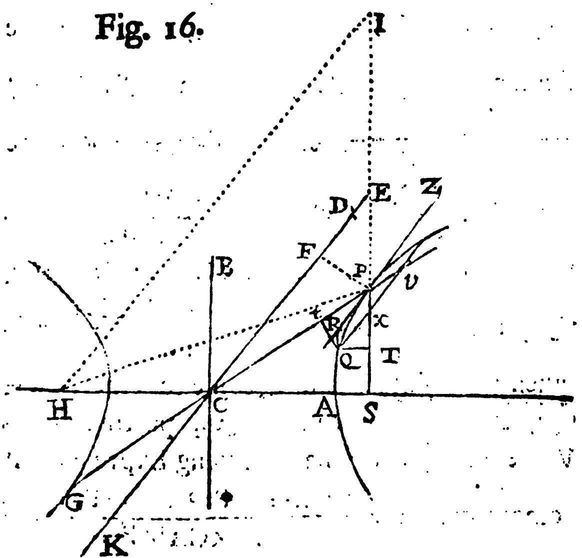
A Body revolving in an Ellipſe, (FIG. XI.) *'tis requir'd to find the Law of the Centripetal Force tending to the* Focus.

HERE the *Lineola* QR is ſuppoſed to be Parallel to Sp, ſo that QR*x*P is a Parallelogram; alſo the *Lineola* QT is a Perpendicular to SP, PF perpendicular to the Conjugate DK; the Points S and H are the *Foci*, and all the reſt as at *Prob.*VI. before. It muſt be premiſed before we proceed to the Inveſtigation with the admirable Author, that EP (cut off by CD, the Conjugate to PG) is equal to AC, half the longer Axe. For drawing IH from the other *Focus* H parallel to DC, we have $SC=CH$ from the Natnre of the *Foci*. and ES EI, becauſe of the Parallels EC and IH, Again, ſince $SP=IP+2EI$, and conſequently $IP+EI$ (that is EP) $=SP-EI$; 'tis plain, that $EP=\frac{SP-EI+IP+EI}{2}=\frac{SP+IP}{2}$. But now, from *Conick Sections*, the Angles HPZ, IPR, are equal, and therefore their Alternates PHI, PIH are equal alſo (for IH parallel to DK, by the *Hypotheſis*, is parallel alſo to the Tangent PZ at the Point P of the Diameter PC, Conjugate to DK.) Therefore PI=PH, and conſequently EP $(=\frac{SP+IP}{2})$ is $=\frac{SP+PH}{2}$, which (from *Conick Sections*) is equal to CA, which is half the longer Axe of the Ellipſe. This premiſed, we proceed to the Inveſtigation of the Law of the Centripetal Force.

The Triangles QTx, PFE are ſimilar (for the Angles T and F are Right ones, and Angle $x=$ Angle E, becauſe of the Parallels Qx and DF) ſo that $QT^{q}=\frac{Qx^{q}\times PF^{q}}{CA^{q}}$; and from the ſimilar Triangles Pxv, PEC, 'tis QR $(=Px)$ $\frac{Pv\times PE}{PC}=\frac{Pv\times AC}{PC}$, becauſe EP=CA. Therefore by *Subſtitution* $\frac{SP^{q}\times QT^{q}}{QR}=\frac{SP^{q}\times Qx^{q}\times PF^{q}\times PC.}{Pv\times CA^{3}}$ But, from *Conick Sections*, $PC^{q}:CD^{q}::Gv^{q}\times Pv^{q}:Qv^{q}$, conſeqnently $Qv^{q}=\frac{Gv\times Pv\times CD^{q}}{PC^{q}}$; alſo when the Points P and Q come together, $Qx=Qv$, therefore inſtead of Qx^{q}, putting in this Value of Qv^{q} in the foregoing Expreſſion, it comes to this, *viz.* $\frac{PS^{q}\times Gv\times Pv\times CD^{q}\times PF^{q}\times PC}{Pv\times CA^{3}\times PC^{q}}$ $=\frac{SP^{q}\times Gv\times CD^{q}\times PF^{q}}{CA^{3}\times PC}=\frac{SP^{q}\times 2CD^{q}\times PF^{q}}{CA^{3}}$, becauſe when the Points P and Q come together, $Gv=2PC$.

$Gv = 2PC$. But from *Conicks* $2CD_q \times PF_q = 2BC^q \times CA^q$, therefore the Expreſſion comes to this, *viz.* $\frac{SP^q \times 2BC^q \times CA^q}{CA^3} = \frac{SP^q \times 2BC^q}{CA} = SP^q \times L$ (becauſe $L = \frac{2BC^q}{CA}$) ſo that the Centripetal Force, which comes to this Expreſſion $SP^q \times L$, dividing by L, a ſtanding Quantity, being the Parameter of the Axe) is reciprocally as SP^q that is, as the Square of the Diſtance from the *Focus*. Q. E. I.

Fig. 16.



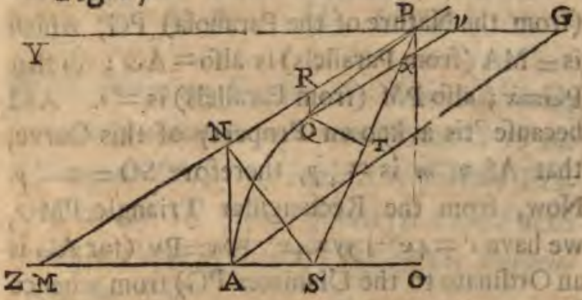
PROB. IX.

A Body revolving in an Hyperbola, 'tis requir'd to find the Law *of the Centripetal Force tending to the* Focus.

HERE AC and CB (*FIG.* XVI.) be the Semiaxes, PG, KD Two Diameters conjugate to each other, Q*t* and PF Perpendeculars to thoſe Diameters reſpectively, Q*v* an Ordinate to PG. The Points S and H are the *Foci*, and from S is drawn SP, cutting the Diameter DK in E, and the Ordinate Q*v* in *x*, and the Figure QRP*x* is ſuppoſed to be a Parallelogram. All the reſt as before in the Ellipſe, and other Figures drawn for the ſame Purpoſe. Now, it muſt be premiſed here alſo, that the Line EP (cut off by KD, the Conjugate to PG) is $=$ AC, the Semi-tranſverſe Axe: For drawing HI from the other *Focus* H, parallel to DC, we have SC=CH from the Nature of the *Foci*; alſo ES is EI, becauſe of the Parallels EC and IH. Farther, ſince $2EI = SP+IP$, and $EI-SP\ (=EP) = IP-EI$; 'tis evident, that EP is $= \frac{IP-EI+EI-SP}{2} = \frac{IP-SP}{2}$.

But now, from *Conick Sections*, the Angles IPZ and HPR are equal, and conſequently their Alternates HIP, IHP are equal alſo (for IH parallel to the Diameter DK, by the *Hypotheſis*, is parallel alſo to the Tangent PZ at the P of the Diameter PC, Conjugate to DK.) Therefore PH = PI, and ſo EP = $\frac{PI - SP}{2}$, is = $\frac{PH - SP}{2}$, which (from *Conick Sections*), is known to be equal to CA, which is the Semi-tranſverſe Axe. This premiſed, all the Steps of the following Proceſs are ſo perfectly the ſame with theſe in the Ellipſe, both as to the Subſtitutions, and the Order and Dependance of one Step upon another, that I need not make the tedious Repetition, but leave the induſtrious Reader to draw it out himſelf, which he cannot miſs of doing, underſtanding but the eaſie Proceſs in the foregoing *Problem*. And the Concluſion here will be, as there, that the Centripetal Force is Reciprocally, as SPq, that is the Square of the Diſtance from the *Focus*. Q. E. I.

Fig. 17.



LEMMA I. (Prob.)

In the Parabola, any Vertex (or Point thro' which the Diameter paſſes) being given, to find the Proportion the Parameter belonging to that Vertex bears to the Diſtance of the ſame Vertex from the Focus.

LET (*FIG.* XVII.) AO be the Axe of the Parabola, PO an Ordinate, S the *Focus*, from which is drawn SP to the Point P, and thro' the ſame the Diameter YPG. Let PM touch the Curve at P, and AG be drawn parallel to PM.

Let $AO=x$. $AG=s$. $SP=z$. $AS=m$. Parameter of the Axe $=p$. Parameter of the Diameter $PG=P$. The Figure AMPG is a Parallelogram, for PG is a Diameter, and therefore parallel to AO, and AG is, by the *Hypotheſis*, parallel to PM. Therefore ſince MA is $=$ AO (from the Nature of the Parabola) PG, which is $=$ MA (from Parallels) is alſo $=$ AO; ſo that PG$=x$; alſo PM (from Parallels) is $= s$. And becauſe 'tis a known Property of this Curve, that AS or m is $= \frac{1}{4}p$, therefore SO $= z - \frac{1}{4}p$. Now, from the Rectangular Triangle PMO, we have $s^2 = 4x^2 + yy = 4x^2 + px = Px$ (for AG is an Ordinate to the Diameter PG) from whence $P = 4x + p$. Again, from the Rectangular Triangle SPO, we have $z^2 = yy + xx - \frac{1}{2}px + \frac{1}{16}pp = \frac{px}{2} + xx + \frac{1}{16}pp$ (putting in for yy its Value px) conſequently $z = x + \frac{1}{4}p$. But P was found $= 4x + p$, therefore $P = 4z$. That is, the Parameter belonging to Diemeter PG, is Quadruple of PS, the Diſtance from the Vertex to the *Focus*. Q. E. I.

The Reſult will be found the ſame, if the Ordinate PO be taken on the other Side the *Focus* S; ſo that SO inſtead of $x - \frac{1}{4}p$, becomes $\frac{1}{4}p - x$.

COR.

It is plain, that PS is $=$ MS, for each is $= x + \frac{1}{4}p$; and therefore alſo the Parameter P is Quadruple of MS.

LEMMA

LEMMA II. (Prob.)

In the foregoing Figure *the Line SN being a Perpendicular from the* Focus *S to the Tangent PM, 'tis requir'd to find the Proportion of SP (the Diſtance from the* Focus *to the Point of Contact P) to the Perpendicular SN.*

ALL the other Symbols ſtanding as before, let SN$=f$. By the *Corollary* of the foregoing *Lemma*, it appears, that PS$=$MS, from whence it follows that PN$=$MN, or PN$=\frac{1}{2}$PM, or PN$^q=\frac{1}{4}$PM$^q=\frac{1}{4}px+xx$. Therefore from the Rectangular Triangle SNP, we have $f^2=x^2+\frac{px}{2}+\frac{1}{16}pp-\frac{1}{4}px-xx=\frac{1}{4}px+\frac{1}{16}pp$. So that $x+\frac{1}{4}p:f::f:\frac{1}{4}p$, that is, f (the Perpendicular from the *Focus* to the Tangent) is a mean Proprotional between the Quantities $x+\frac{1}{4}p$ and $\frac{1}{4}p$, that is between the Diſtances of the Point of Contact, and of the principal Vertex from the *Focus*, or in other Terms, that AS SN :: SN : SP. Q. E. I.

COR.

Hence becauſe of $\frac{1}{4}p$, or AS a ſtanding Quantity, f^2, or SN^q, is always as $x+\frac{1}{4}p$, or SP: That is, the Diſtance from the *Focus* PS is ever as the Square of the Perpendicular from the *Focus* to the Tangent.

PROB. XI.

A Body revolving in a Parabola, 'tis requir'd to find the Law of the Centripetal Force tending to the Focus.

THE reſt of the Conſtruction remaining as in the foregoing *Lemma*, let QR be parallel to SP, QT perpendicular to the ſame, Qv parallel to the Tangent PM, cutting SP in x, and YPG in v. Before I proceed to the Inveſtigation, 'tis neceſſary to premiſe, that Pv is Px, which is thus prov'd. The Angle PMS (= MPY, as being Alternates) is = Angle Pvx, becauſe of the Parallels MS and PG; alſo the Angle vPx is = PSM, becauſe of Parallels. Therefore the Triangles MPS and Pxv are ſimilar, and ſo $MS : SP :: Px : Pv$; but by *Cor.* of *Lemma* I. foregoing, PS is = MS; therefore $Px = Pv$. This premiſed, let the Parameter of the Diameter PG be call'd L. Then (from the Nature of the Parabola) $L\times Pv = Qv^q$ $=4SP\times Pv$ (becauſe $L=4SP$, by *Lemma* I. foregoing) $=4SP\times Px$ (becauſe $Pv = Px$, as was juſt now ſhewn) $=4SP\times QR$ (becauſe the Figure $QRPx$ is a Parallelogram, by the Conſtruction.) But when the Points P and Q come together, the *Lineola* Qv will be coincident with or equal to Qx; therefore in that Caſe Qx^q ſhall $= 4SP\times QR$. *Laſtly*, Becauſe the Triangles QxT, SPN are ſimilar (for the Angles QTx, and SNP are Right ones, and $QxT=NPS$ from Parallels (therefore $Qx^q : QT^q :: PS^q : SN^q$; but (by *Lemma* II. foregoing) 'tis $PS : SN :: SN : AS$, and conſequently $PS^q : SN^q :: PS : AS$; therefore (by Equality of Proportion) $Qx^q : QT^q :: PS : AS$, and (multiplying PS and AS by 4 QR) 'tis $Qx^q : QT^q :: 4SP\times QR : 4AS\times QR$. But $Qx^q = 4PS\times QR$; therefore $QT^q = 4AS\times QR$. Therefore ſubſtituting this Value of QT^q in the general Expreſſion $\frac{SP^q\times QT^q}{QR}$, it amounts to $SP^q\times 4AS$, and $4AS$ being = the Parameter of the Axe, and ſo a ſtanding Quantity, it comes to SP^q. So that the Centripetal Force is Reciprocally as SP^q; that is, as the Square of the Diſtance from the *Focus*. Q. E. I.

COROLARIES *to the preceding* PROBLEMS *concerning the Law of the Centripetal Force tending to the* Foci *of the* Conick Sections.

COR. I.

It follows, that if a Body moves from any Place, as P, and is drawn by a Centripetal Force Reciprocally, proportional to the Square of the Diſtance from the Center to which thoſe Forces are directed; that then it moves in a *Conick Section*, whoſe *Focus* is coincident with that Center.

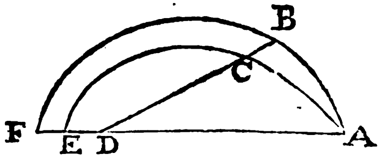
COR. II.

And if the Velocity with which the Body goes out of the Place P be ſuch, that while the *Lineola* PR is deſcribed by it, the Centripetal Force carries it thro' the *Lineola* QR in the ſame Time, then it moves in a *Conick Section*, whoſe Parameter is $= \frac{QT^q}{QR}$, when the Points P and Q come together.

And, with reſpect to the particular *Sections*, from the Law found we may infer,

COR.

Fig. 18.



Cor. III.

That if Two Ellipſes, as FBA, ECA (*FIG.* 18.) have their Vertex A, and one *Focus* (as the Point D) common to them both, that then the Centripetal Force in C ſhall be to that in B (the *Focus* D being the Center of the Forces) as BDq : DCq. For putting the Force in A$=$F, that in C$=f$, that in B$=\varphi$; 'tis plain, that $\frac{f}{\varphi}=\frac{f}{F}\times\frac{F}{\varphi}$, but $\frac{f}{F}=\frac{ADq}{CDq}$, and $\frac{F}{\varphi}=\frac{BDq}{ADq}$, therefore $\frac{f}{\varphi}=\frac{BDq}{CDq}$, and ſo $f:\varphi::$ BDq : CDq.

That the Two Ellipſes may be ſuch as is requir'd, the Lines FD, ED muſt be in the duplicate *Ratio* of the ſhorter Axes, the Mention of which is ſufficient.

Cor. IV.

In the Parabola, the Centripetal Forces are Reciprocally, as the Squares of the Parameter of the Axe encreaſed by Quadruple the Abſciſs (taken in the Axe) for any Point of the Curve. That is (in the Terms of *Lemma* I. foregoing) the Centripetal Forces are Reciprocally as $16xx+8px+pp$. Which is manifeſt, becauſe by the Law found, the Centripetal Force is Reciprocally as the Square of the Diſtance from the *Focus*; but (by the foremention'd *Lemma*) the Expreſſion of any Diſtance from the *Focus* is $x+\frac{1}{4}p$. Therefore, *&c.*

'Tis alſo equally true, that the Centripetal Forces are Reciprocally as the Squares of the Parameters belonging to the ſeveral Vertices, or Points where the Body is: For thoſe Parameters are Quadruple the reſpective Diſtances from the *Focus*.

SCHOL.

The Law of the Centripetal Force tending to the *Focus* of the Ellipſe being diſcover'd, the Application may eaſily be made to the reſt of the *Conick Sections* without a particular Inveſtigation, as the celebrated Author himſelf obſerves. (See *FIG.* XI.)

If the *Focus* S of the Ellipſe to which the Centripetal Force tends, continues together with the Vertex B, while the other *Focus* H removes to an infinite Diſtance, the Ellipſe now is converted into a Parabola; but the Body will ſtill move into a Perimeter of this Curve, and the Law of the Centripetal Force tending to the *Focus* S continues what it was before, *viz.* Reciprocally as the Square of the Diſtance from S.

Suppoſing all as before, if the *Focus* H moves backwards, ſo as to come behind the Vertex B, both the Vertices A and B, lying now between the *Foci* S and H, the Ellipſe becomes an Hyperbola, and the Body moving in the Perimeter of this Curve, the Law of the Force tending to the *Focus* S continues the ſame as at firſt.

If the *Focus* H and Vertex A be ſuppoſed fix'd, and S, to which the Forces tend, removes to an infinite Diſtance, the Ellipſe is changed into a Parabola, and the Forces tending to a Point infinitely diſtant are all equal.

The *Focus* H and Vertex A continuing, if the *Focus* S, to which the Forces tend, moves backwards, as in *Caſe* II. then we have an Hyperbola again, in the Periphery of which the Body ſhall ſtill move, but with a Centrifugal Force (inſtead of a Centripetal) which ſhall be according to the eſtabliſh'd Law, Reciprocally as the Square of the Diſtance from S.

If the Two *Foci* S, H come together, the Ellipſe becomes a Circle, and the Force tending to the Center is equable.

If the *Foci* S, H coincide with the Vertices B, A, the Ellipſe degenerates into a Right Line; in which Caſe there's no Centripetal Force tending to any Point without the Line it ſelf.

THEOR. XI.

Bodies revolving about a common Center, and the Centripetal Force being in a Reciprocal duplicate Ratio *of the Diſtances from the Center; then the* Latera Recta *of the Orbs deſcrib'd ſhall be in the duplicate* Ratio *of the* Area'*s which the Bodies deſcribe in the ſame Time, by Rays drawn to that Center.*

FOR (by *Cor.* II. of *Prob.* XI.) $L = \frac{QT^q}{QR}$; but QR in a given Time is as the Centripetal Force (by the Second *Law of Motion*) that is, as SP_q Reciprocally (by *Prob.* IX.) Therefore $\frac{QT^q}{QR}$ is as $QT^q \times SP^q$, that is, L is as $QT^q SP^q$; but the *Area* is as $QT \times SP$. *Ergo*, &c. Q. E. D.

COR.

The whole *Area's* of Ellipſes are in the *Ratio* compounded of the $\frac{1}{2}$ *Ratio* of the *Latera Recta*, and the *Ratio* of the Periodick Times

For thoſe *Area's* are in the *Ratio* compounded of the *Ratio* of the Portions deſcrib'd in the ſame Time, and that of the Periodick Times: But by this *Prop.* thoſe Portions are in the ſubduplicate *Ratio* of the *Latera Recta*. Therefore, &c.

And ſince (by *Prop.* 191. of *Gregory St. Vincent*) the *Area's* of Ellipſes are as the Rectangles under their Axes, it's certain, that thoſe Rectangles are alſo in the ſame *Ratio*.

THEOR. XII.

Suppoſing all as before, the Periodick Times in Ellipſes are in the $\frac{3}{2}$ *(the Seſquiplicate)* Ratio *of the Tranſverſe Axes.*

FOR reaſſuming the Symbols as in *Theor.* 8. we find that $T : t :: A \times n : a \times N$; but $A : a :: D \times B : d \times b$ (by the foremention'd *Theor.* of St. *Vincent*) therefore $T : t :: D \times B \times n : d \times b \times N$. But $D : d :: L^{\frac{1}{2}} \times B^{\frac{1}{2}} : l^{\frac{1}{2}} \times b^{\frac{1}{2}}$, from *Conicks*; and $n : N :: l^{\frac{1}{2}} : L^{\frac{1}{2}}$, by the foregoing *Theorem*, therefore $T : t :: L^{\frac{1}{2}} \times B^{\frac{1}{2}} \times B \times l^{\frac{1}{2}} : l^{\frac{1}{2}} \times b^{\frac{1}{2}} \times b \times L^{\frac{1}{2}}$, that is (dividing by $L^{\frac{1}{2}} \times l^{\frac{1}{2}}$) $T : t :: B^{\frac{3}{2}} : b^{\frac{3}{2}}$. The Periodick Times in Ellipſes are in the Seſquiplicate *Ratio* of the Tranſverſe Axes. Q. E. D.

COR.

COR.

The Periodick Times in Ellipſes are the ſame with thoſe in Circles, whoſe Diameters are equal to the longer Axes. For by the Converſe of *Caſe* IV. from the *Corollaries* of *Theor.* III. this Proportion of the Squares of the Times and the Cubes of the Rays belongs to Circles, when the Law of the Centripetal Force is the ſame that 'tis here, *viz.* Reciprocally as the Square of the Diſtance from the Center to which the Forces are directed.

THEOR. XIII.

The Centripetal Force tending to the Focus, *as before*; *PR being a Tangent at P, the Place of the Body, and ST a Perpendicular from the* Focus *to it, the Velocity of the Body in P ſhall be in the* $\frac{1}{2}$ Ratio *of* $\frac{ST^q}{L}$ *Reciprocally.* (*See* FIG. XI.)

FOR the Velocity is as the little Ark PQ deſcrib'd in a given Time; that is, as the Tangent PR, or becauſe of Parallels, as Qx. But, from ſimilar Triangles, QTx, SYP, Qx

or $PR = \frac{SP \times QT}{SY}$; therefore PQ is as SY Reciprocally, and $SP \times QT$ directly; that is, as SY Reciprocally, and the *Area* deſcrib'd in a given Time directly; but (by *Theor.* XI.) $SP^q \times QT^q$ is as L, therefore PQ is as $\frac{L^{\frac{1}{2}}}{SY}$, that is, in the $\frac{1}{2}$ *Ratio* of $\frac{SY^q}{L}$ Reciprocally. Q. E. D.

In order to the Inveſtigation of the many noble *Corollaries* that follow from this *Theorem*, let theſe perpetual Symbols ſtand.

SP (the Diſtance from the *Focus* to any Point in the Curve $= z$. SY (the Perpendicular from the *Focus* to the Tangent at the ſame Point) $= y$. The Velocity in a *Conick Section* at the Diſtance z, put $= V$. The Velocity in a Circle at the ſame Diſtance z, put $= W$. The Velocity in a Circle (whoſe *Radius* is equal to any other Diſtance, whether greater or leſs than z) put $= w$. The longer Axe of any Ellipſe put $= 2B$, the ſhorter $= 2D$, the Parameter $= L$; and this Symbol L I uſe alſo for the Parameter of any *Conick Section* in general, as the Occaſion requires.

Now then, the Expreſſion of the Velocity in a *Conick Section* (at any Diſtance as z) is $\frac{L^{\frac{1}{2}}}{y}$; that is, V is ever as $\frac{L^{\frac{1}{2}}}{y}$. Likewiſe the Expreſſion of the Velocity in a Circle at the Diſtance z wou'd be $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$. For the *Radius* being $= z$, then L in the Circle is $= 2z$, or the Diameter; and y the Perpendicular from the *Focus* to the Tangent, being in the Circle = the *Radius*, that is $= z$: Hence in the Terms of the Circle $\frac{L^{\frac{1}{2}}}{y}$ is $\frac{\sqrt{2z}}{z}$, or $\frac{\sqrt{2} \times z^{\frac{1}{2}}}{z} = \frac{\sqrt{2}}{z^{\frac{1}{2}}}$. So that then W is as the Expreſſion $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$, or $\sqrt{\frac{2}{z}}$. Now for the *Corollaries* themſelves.

COR. I.

Becauſe V is as $\frac{L^{\frac{1}{2}}}{y}$, therefore $L^{\frac{1}{2}}$ is as Vy, and L is as V^2y^2. That is, the Parameters of the Orbs are in the *Ratio* compounded of the duplicate *Ratio* of the Velocities, and the duplicate *Ratio* of the Perpendiculars from the *Focus* to the Tangents.

COR. II.

When the Quantity y (in the Expreſſion of the Velocity) becomes coincident with the greateſt or leaſt Diſtance from the *Focus*; 'tis manifeſt then, that in thoſe Points of greateſt or leaſt Diſtance, the Velocities are as thoſe Diſtances inverſly, and the ſquare Roots of the Parameters directly.

COR. III.

Becauſe V is as $\frac{L^{\frac{1}{2}}}{y}$, and W as $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$, therefore if $y = z$ (which cannot be but when z denotes the greateſt or leaſt Diſtance) then the Expreſſions are $\frac{L^{\frac{1}{2}}}{z}$, and $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$; which (multiplying both by $z^{\frac{1}{2}}$) comes to $L^{\frac{1}{2}}$, and $\sqrt{2} \times z^{\frac{1}{2}}$, or $\sqrt{L}$, and $\sqrt{2z}$; ſo that $V : W :: \sqrt{L} : \sqrt{2z}$. That is, the Velocity in a *Conick Section*, at the greateſt or leaſt Diſtance from the *Focus*, is to the Velocity in a Circle at the ſame Diſtance, in the ſubduplicate *Ratio* of the Parameter to that Diſtance doubled.

COR.

C o r. IV.

Let $y=D$ the ſhorter Semiaxe of an Ellipſe, then the Expreſſion of the Velocity for an Ellipſe at the Extremity of the ſhorter Axe, that is, at the mean Diſtance from the *Focus*, is $\frac{L^{\frac{1}{2}}}{D}=L^{\frac{1}{2}}-\!\!:\frac{L^{\frac{1}{2}}\times B^{\frac{1}{2}}}{\sqrt{2}}=\frac{\sqrt{2}}{B^{\frac{1}{2}}}$. And the Expreſſion for the Circle (being in general $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$) comes (if z be put $=B$) to $\frac{\sqrt{2}}{B^{\frac{1}{2}}}$. So that in this Caſe V is $=$ W, ſince the Values of both are the ſame. That is, the Velocities of Bodies (revolving in Ellipſes) at the mean Diſtances from the common *Focus*, are equal to the Velocities in Circles at the ſame Diſtances.

C o r. V.

Since V is always as $\frac{L^{\frac{1}{2}}}{y}$, therefore where L continues ſtill the ſame, V is as $\frac{1}{y}$. That is, in the ſame, or equal Figures, or in unequal Figures whoſe Parameters are equal, the Velocity is Reciprocally as the Perpendicular from the *Focus* to the Tangent.

COR. VI.

The Expreſſion $\frac{L^{\frac{1}{2}}}{y}$, in the Parabola comes to $\frac{L^{\frac{1}{2}}}{z^{\frac{1}{2}}}$, becauſe (by *Cor.* to *Lemma* II. forgoing) the Perpendicular from the *Focus* to the Tangent, *viz.* y is ever in the ſubduplicate *Ratio* of z, the Diſtance from the *Focus* to the Point of Contact. Or becauſe in ſpeaking of one and the ſame Figure, L· is a ſtanding Quantity, therefore V is ever as $\frac{1}{z^{\frac{1}{2}}}$. That is, in the Parabola the Velocities are in the ſubduplicate *Ratio* of the Diſtances from the *Focus* Reciprocally. In the Hyperbola they will be in more than this *Ratio*, and in the Ellipſe in leſs; the Reaſon of which is the different Proportion of the Perpendiculars to the Diſtances in thoſe Curves.

COR. VII.

The Expreſſion for any *Conick Section* being $\frac{L^{\frac{1}{2}}}{y}$, and that for a Circle (whoſe *Radius* is $= z$) being $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$; 'tis plain then (whatever Diſtance z repreſents, whether equal to that in the *Conick Section*, or greater or leſs than it) that the Velocities are as $\frac{L^{\frac{1}{2}}}{y} : \frac{\sqrt{2}}{z^{\frac{1}{2}}}$, that is, as $L^{\frac{1}{2}} \times z^{\frac{1}{2}} : \sqrt{2} \times y$, or as $\sqrt{L \times z} : \sqrt{2yy}$, that is, as $\sqrt{\frac{1}{2}L \times z} : y$. Now $\sqrt{\frac{1}{2}L \times z}$ is a mean Proportional between $\frac{1}{2}$ the Parameter of the *Conick Section*, and z the *Radius* or Diſtance in the Circle. So that this general Concluſion is to be inferr'd, that the Velocity in a *Conick Section* at any Diſtance from the *Focus*, is to the Velocity in a Circle at *any other Diſtance* (or whoſe *Radius* is = to *any other Diſtance* from the *Focus*) as a mean Proportional between that *other Diſtance* or *Radius*, and half the Parameter, is to a Perpendicular drawn from the *Focus* to the Tangent at the Point of Diſtance in the *Conick Section*. That is, z being the Diſtance in (or *Radius* of) a Circle, in which the Velocity is expreſs'd by w, and V being the Velocity in a *Conick Section* at any Diſtance, equal or unequal to the former; the Concluſion in our Symbols ſtands thus, $V : w :: \sqrt{\frac{1}{2}L \times z} : y$. Now, from this general Concluſion, all that relates to the Proportion of the Velocities in *Conick Sections* and Circles, may eaſily and naturally be inferr'd.

As, 1. If z, or the *Radius* of the Circle, be equal to the Diſtance (from the *Focus*) in the *Conick Section*; then it follows, that the Velocity in a *Conick Section* is to the Velocity in a Circle at the *ſame Diſtance*, as a mean Proportional between that *ſame Diſtance* (or *Radius*) and $\frac{1}{2}$ the Parameter, to a Perpendicular from the *Focus*, to the Tangent at the Point of Diſtance in the *Conick Section*. This holds in all the *Conick Sections*. But the reſt that follow, all except one, referring particularly to the Parabola, I note here, that in this Curve the Expreſſion of the Diſtance from the *Focus* to any Point in the Curve is $= \frac{1}{4} L + x$ (x being the correſpondent Abſciſs;) as alſo, that the Quantity y, the Perpendicular from the *Focus* to the Tangent at any Point, is $\sqrt{\frac{1}{16} L^2 + \frac{1}{4} Lx}$; both which are plain from the *Calculus* in the 2d *Lemma* to *Prob*. XI. Therefore to proceed,

2. In the Parabola, if we put $z = \frac{1}{4} L + x$; then $Lz = \frac{1}{4} L^2 + Lx$; but $yy = \frac{1}{16} L^2 + \frac{1}{4} Lx$, therefore in this Caſe $Lz = 4yy$, and $\frac{1}{2} Lz : yy :: 2 : 1$, and $\sqrt{\frac{1}{2} Lz} : y :: \sqrt{2} : 1$, or $V : w :: \sqrt{2} : 1$. That is, the Velocity in a Parabola at any Diſtance from the *Focus*, is to the Velocity in a Circle at the *ſame Diſtance*, as $\sqrt{2} : 1$. *Note*, I ſay, at the *ſame Diſtance* in the Parabola for here $\frac{1}{4} L + x$ is put $=$ to z, the *Radius* of the Circle.

3. If in the Parabola we put $z = y = \frac{1}{4}L$: Then $\sqrt{\frac{1}{2}Lz} : y :: \sqrt{2} : 1$, and conſequently $V : w :: \sqrt{2} : 1$, in this Caſe alſo. That is, the Velocity in a Parabola at the *leaſt Diſtance* from the *Focus* is to the Velocity in a Circle at the *ſame Diſtance*, as $\sqrt{2} : 1$. Note again here, I ſay, the *leaſt Diſtance*; becauſe 'tis put $=$ to $\frac{1}{4}$ of the Parameter, or becauſe the Diſtance is ſuppoſed to be coincident with the Perpendicular from the *Focus* to the Tangent, which cannot be but when 'tis the leaſt. Alſo z, the *Radius* of the Circle, is here ſuppoſed equal to the Diſtance in the *Section*.

4. In any *Conick Section*, if we put $z = \frac{1}{2}L$: Then $\sqrt{\frac{1}{2}Lz} = \frac{1}{2}L$; conſequently $\sqrt{\frac{1}{2}Lz} : y :: \frac{1}{2}L : y$, or $V : w :: \frac{1}{2}L : y$. That is, the Velocity any where in a *Conick Section*, is to the Velocity in a Circle at the Diſtance of $\frac{1}{2}$ the Parameter, as $\frac{1}{2}$ the Parameter is to a Perpendicular from the *Focus* to the Tangent at the Point of Diſtance in the *Conick Section*. This is general for all the *Conick Sections*, there being nothing ſuppoſed in the *Calculus* that does reſtrain or limit the Property to any one Particular.

5. If in the Parabola we ſuppoſe $z = \frac{1}{8}L + \frac{1}{2}x$. Then $Lz = \frac{1}{8}L^2 + \frac{1}{2}Lx$, and conſequently $\frac{1}{2}Lz = \frac{1}{15}L^2 + \frac{1}{4}Lx$, which is yy (as we hinted before.) Therefore $\sqrt{\frac{1}{2}Lz} = y$, and therefore $V = w$. That is, the Velocity in a Parabola is every where equal to the Velocity in a Circle at $\frac{1}{2}$ the

Diſtance, or in a Circle whoſe *Radius* is $\frac{1}{2}$ the Diſtance from the *Focus*, in the Parabola. Note, I ſay, *whoſe* Radius *is*$=$*to* $\frac{1}{2}$ *the Diſtance in the Parabola*; for the Diſtance being univerſally expreſſed by $\frac{1}{4}L+x$, I here put the *Radius* $z=\frac{1}{8}L+\frac{1}{2}x$.

And univerſally, if we we make $z : \frac{L}{4}+x :: n : 1$, and ſo $z=\frac{nL}{4}+nx$. Then will $\sqrt{\frac{1}{2}Lz}=\sqrt{\frac{nL^2}{8}+\frac{nLx}{4}}$; and conſequently univerſally $V : w :: \sqrt{\frac{nL^2}{8}+\frac{nLx}{4}} : y$. That is, the Velocity in the Parabola will be to the Velocity in a Circle (whoſe *Radius* is n times the Diſtance in the Parabola) as $\sqrt{\frac{nL^2}{8}+\frac{nLx}{4}} : y$. From whence particular Caſes may be deduced at Pleaſure.

And thus much for this admirable *Theorem* relating to the Velocities of Bodies, together with its *Corollaries*.

PROB. XII.

Any Point being taken at Liberty in the Axe of an Ellipſe, 'tis requir'd to find the Law of the Centripetal Force tending to the ſame. [See *FIG.* XIX. after the *Preface*.)

LET the Points s. s. be taken of either Side the *Focus* of the Ellipſe, which I make to be S. Then let the Line sP be drawn, and QT let fall Perpendicular to it, and Qx parallel to the Tangent PR, interſecting sP in the Point x. *Laſtly*, Let QR be parallel to sP, and the Interſection of sP with the Conjugate DC, be in the Point O. This being ſuppoſed with all the reſt, as at *Prob.* IX. we have here the Triangles QTx and PPO, ſimilar to one another, as there the Triangles QTx and FPE were ſimilar. So the QT^q is $= \frac{Qx^q \times PF^q}{PO^q}$. But the laſt *Ratio* of Qx to the Ordinate Qv, is a *Ratio* of Equality; conſequently inſtead of Qx^q we may put Qv^q, and ſo QT^q (by ſubſtituting the Value of $Qv^q = \frac{Gv \times Pv \times CD^q}{PC^q}$, from *Conicks*) will $= \frac{PF^q \times Pv \times vG \times CD^q}{PO^q \times PC^q}$. Again, the Triangles Pxv and POC are ſimilar; therefore $Px = \frac{Pv \times PO}{PC}$; conſequently QR, which is $= Px$ from Parallels, is to be expreſs'd by this Value. Therefore the Centripetal Force, which is Reciprocally as $\frac{sP^q \times QT^q}{QR}$, is as $\frac{sP^q \times PF^q \times Pv \times vG \times CD^q \times PC}{Pv \times PO^3 \times PC^q}$. And putting $BC^q \times CA^q$ (which is a ſtanding Quantity) inſtead of $PF^q \times CD^q$, and inſtead of vG putting $2PC$; by due Reduction the Expreſſion comes to $\frac{sP^q}{PO^3}$, which gives the Law of the Centripetal Force, *viz.* that 'tis Reciprocally, as the Square of the Diſtance from the Point s, divided by the Cube of the Line PO, intercepted between the Tangent PR and the Line DC, which is Conjugate to PC. From hence,

C o r. I.

If the Point s coincide with S, the *Focus* of the Ellipſe; then the Line PO will alſo be coincident with PE, which is equal to AC, the Semi-tranſverſe Axe. And therefore $\frac{sP^q}{PO^3}$ will in this Caſe come to $\frac{SP^q}{CA^3}$; or becauſe of CA^3, a ſtanding Quantity, it will be reduced to SP^2. So that the Law of the Centripetal Force tending to the *Focus*, is this, that 'tis Reciprocally as the Square of the Diſtance from the *Focus*.

C o r. II.

If the Point s coincide with C, the Center of the Ellipſe; then sP and PO alſo do both coincide with the Line PC, which is the Semi-Diameter for the Point P. Therefore the Expreſſion $\frac{sP^q}{PO^3}$ comes in this Caſe to $\frac{PC^q}{PC^3}$, that is, to $\frac{1}{PC}$. So that the Law of the Centripetal Force tending to the Center of the Ellipſe, is this, that 'tis directly as the Diſtance from the Center. And both theſe Laws are the ſame with thoſe found by particular Proceſſes before.

I

I need not ſay, that this done for the Ellipſe, may eaſily (*mutatis mutandis*, according to the Nature of the Curve) be applied to the Hyperbola; ſo that there's no Need of another Operation for that. But becauſe the Application to the Parabola may not be quite ſo obvious, I think it proper to do that by it ſelf.

PROB. XIII.

Any Point being taken at Liberty in the Axe of a Parabola, 'tis requir'd to find the Law of the Centripetal Force tending to that Point. (See *FIG.* XX. after the *Preface.*)

LET the Point S be taken any where in the Axe AS; thro' which Point S let SO be drawn parallel to the Tangent PR, PO parallel to the Axe AS, and conſequently a Diameter for the Point P. Q*v* an Ordinate to PO, and conſequently parallel to PR, meeting alſo SP in *x*, and PO in *v*. *Laſtly*, Let QR be parallel to SP, and the Lines QT and PF Perpendiculars from the Points Q and P, to the Lines SP and SO. This premiſed, the Triangles QT*x* and PSF are ſimilar. Therefore $QT = \frac{Qx^q \times PF^q}{PS^q}$. But the laſt *Ratio* of Q*x* to Q*v* (that is, when the P and Q come together) is a *Ratio* of Equality; ſo that then $QT^q = \frac{Qv^q \times PF^q}{PS^q}$. And if L be the Parameter belonging to the Vertex

P, then $L\times Pv = Qv^q$, and ſo $QT^q = \frac{L\times Pv\times PF_q.}{PS_q}$ But QR is = Px (from Parallels) = $\frac{Pv\times PS}{PO}$ (from the ſimilar Triangles Pxv and PSO.) Therefore $\frac{QT^q}{QR} = \frac{L\times PF^q\times PO}{PS^3}$; and conſequently $\frac{SP^q\times QT^q}{QR}$ (which is Reciprocally as the Centripetal Force) is alſo as the Quantity $\frac{L\times PF^q\times PO.}{PS}$ And this is the *Law* of the Centripetal Force tending to the Point S taken any where in the Axe.

C o r.

Therefore if the Point S coincide with the *Focus* of the Parabola, PS will be=PO; for in that Caſe PS will = SG from the Nature of the Parabola, but SG is = PO from Parallels, therefore PS=PG. So that the Expreſſion $\frac{L\times PF^q\times PO}{PS}$ comes in this Caſe to $L\times PF^q$. But SL being a Perpendicular from the *Focus* to the Tangent, PF is equal to it, from Parallels; and therefore PF^q is ever as SP, from the Nature of a Parabola. So that $L\times PF_q$ is ſtill as $L\times SP$. But alſo from the Nature of the Parabola, L is = 4 SP; therefore $L\times SP$ is ever as SP^q. So that the Centripetal Force (tending to the *Focus*) is ever as the Square of the Diſtance from it, Reciprocally.

I am aware that that there may be more general Inveſtigations of the Laws of the Centripetal Force than theſe, and I am acquainted with ſome of them; but what I have here ſaid is ſufficient for my Purpoſe.

FINIS.